Exercises for lecture „Introduction to nuclear physics“

Important constant:

Electron rest energy: 0.511 MeV
Proton rest energy: 938 MeV
Neutron rest energy 939.6 MeV
Mass unit energy: 931.5 MeV
Deuteron rest energy: 1875.6 MeV
Alpha rest energy: 3727.4 MeV

$\hbar c$ = 197,3 MeV·fm = 197,3 eV·nm

Hyperfine structure constant: $$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137}$$

Classical electron radius: $$r_0 = \frac{e^2}{4\pi\varepsilon_0 m_e c^2} = \frac{\alpha \cdot \hbar c}{m_e c^2} = 2.82\text{fm}$$

Avogadro constant: $6.022 \cdot 10^{26}$ kmol^{-1}

Velocity in the unit of light velocity

1) Introduction (work with units and quantities, review relativity foundations, simple fundamental quantum properties)

1) Millikan experiment. Determine electron charge from Millikan experiment data. We know from Millikan Nobel lecture, that mean time of fall of droplet of ricin oil in gravitation through path 1.303 cm was 120.35 s. The four groups of moving up were seen after switch on of electric field with intensity 6000 V/cm through path 1.303 cm: 67.73 s, 26.40 s, 16.50 s a 11.90 s. Needed constants are: air density 1.29 kg/m^3, density of ricin oil 960 kg/m^3. Dynamical viscosity of air $1.83 \cdot 10^{-5}$ Pa·s. [R = 0.972 μm, v_1 = 0.108 mm/s, v_2 = 0.192 mm/s, 0.494 mm/s, 0.790 mm/s a 1.094 mm/s, Q = $1.676 \cdot 10^{-19}$ C, $3.362 \cdot 10^{-19}$ C, $5.015 \cdot 10^{-19}$ C a $6.713 \cdot 10^{-19}$ C]

2) Helium nucleus has kinetic energy 98 MeV. Determine its velocity, momenta and de Broglie wave length. Which difference is between velocities determined by relativistic equations and by nonrelativistic approximation? [v = 0.229 c, p = 860.3 MeV/c, λ = 1.43 fm, nonrelativistic is by 1.96 % higher]

3) Proton has kinetic energy 570 MeV. Determine its velocity, momenta and de Broglie wave length. Which difference is between velocities determined by relativistic equations and by the nonrelativistic approximation? [v = 0.783 c, p = 1180.8 MeV/c, λ = 1.05 fm, nonrelativistic is 1.1 c]

4) Which is wave length and frequency of electromagnetic radiation emitted by photons with energy 2 eV, 40 keV and 1.5 MeV. Which number of photons is emitted by source with power 10 W with such spectrum? [λ = 618.6 nm, 0.0309 nm, 0.824 pm, ν = 0.484 PHz, 9.7 EHz, 0.364 ZHz, N = $3.12 \cdot 10^{19}$ s^{-1}, $1.56 \cdot 10^{15}$ s^{-1}, $4.16 \cdot 10^{13}$ s^{-1}]

5) Which is energy of excited states of hydrogen atom? Which are velocity and kinetic energy of electron? Which wave length light emitted during transition between third and second excited states? Use Bohr model of atom. [$E_K = 13.61 \cdot (1/n^2)$ eV, $v = 0.00730 \cdot (1/n)$ c, $\lambda = 655.5$ nm]

6) Electrons with energy 20 eV are scattered on crystal lattice with lattice constant 6 nm. In which distance from original direction of electrons will be first interference maximum on photographic plate in the distance 2 m? [$\lambda = 0.274$ nm, $\sin \vartheta = 0.02283$, x = 4,57 cm]

7) Neutron half life is 15 minutes. Which is velocity, total energy and kinetic energy in the coordinate system, where neutron decay half life is 45 min. Determine de Broglie wave length of this neutron. [v = 0.943 c, E = 2818.8 MeV, $E_K = 1879.5$ MeV, $\lambda = 0.465$ fm]

8) Meson π^+ (m = 139.6 MeV/c^2) decays dominantly to muon and muon antineutrino. Its mean life time is 26 ns. Which will be its mean life time in the laboratory, if its kinetic energy is 960 MeV. Which will be its velocity, momenta and de Broglie wave length? Which velocity will have muon (m = 105.66 MeV/c^2) emitted in the same direction and in the opposite direction against original movement of π^+ meson in laboratory and center of mass, if we know that muon in the center mass system is 4.1 MeV? Prove, that it is really this energy. [$\tau = 204.8$ ns, v = 0.9919 c, p = 1090 MeV/c, $\lambda = 1.13$ fm, v (center mass system) = 0.271 c, v (laboratory in the same direction) = 0.9953 c, v (laboratory in the opposite direction) = 0.9859 c, decay energy Q = 33.94 MeV]

9) Which wave lengths emits hydrogen like ion of helium? [$E_K = 54.44 \cdot (1/n^2)$ eV, v = $0.0146 \cdot (1/n)$ c, λ(to ground state) = 30.3 nm, 25,6 nm, … λ(to first excited) = 164 nm, 121,3 nm …]

10) Nucleus of uranium 235 accelerated by means of heavy ion accelerator to the kinetic energy 2.5 GeV/A decays by alpha decay. The alpha particle has in this case the centre of mass system kinetic energy 4.5 MeV. Which will be velocity of uranium nucleus and its de Broglie wave length? Which will be alpha particle velocity in the coordinate system connected to the nucleus and in the laboratory, if it will be emitted in the direction of uranium movement, against its movement and perpendicularly to uranium movement? Which will be kinetic energy, momenta and de Broglie wave length of alpha particle. [v = 0.9625 c, λ = 0.0016 fm, v (from the view of nucleus) = 0.0491 c, $E_K = 4.5$ MeV, p = 183.2 MeV/c, $\lambda = 6.8$ fm, v (emitted to original direction) = 0.9659 c, $E_K = 10665.2$ MeV, p = 13900.8 MeV/c, λ = 0.089 fm, v (emitted against original direction) = 0.9587 c, $E_K = 9374.9$ MeV, p = 12560 MeV/c, $\lambda = 0.099$ fm, v(emitted perpendicularly to the original direction) = 0,9637 c, E_K = 10230 MeV, p = 13450 MeV/c, λ = 0,092 fm]

11) Which must be energy of electron "to see" atom (size approximately 0.1 nm), to nucleus with size 10 fm „to see" structures inside nucleus with sizes 1 fm, 0.1 fm a 0.01 fm. Which is velocity and momenta of such electron. [$E_K = 0.15$ keV, 123 MeV, 1.24 GeV, 12.4 GeV a 124 GeV, v = 0.024 c, 0.9999914 c, 0.999999915 c, 0.99999999915 a 0.9999999999915 c, p = 0,124 MeV/c, 124 MeV/c, 1.24 GeV/c, 12.4 GeV/c a 124 GeV/c]

12) Which must be energy of alpha particle „to see" gold nucleus with size approximately 14 fm. Which will be alpha particle momenta and velocity? Which will be path radius of this

alpha particle in magnetic field 0.03 T. [E_K = 1.04 MeV, p = 88.4 MeV/c, v = 0.023 c, R = 4.9 m]

13) Kinetic energy of proton is determined by means of time of flight in given experiment. Which is values of energy for protons, which fly over distance 4 m during 15, 19, 25 a 35 ns? Which are moments of such protons. Which are de Broglie wave lengths? Discuss, which of them are relativistic and which are nonrelativistic? [E = 2047.5 MeV, 1316.6 MeV, 1108.9 MeV a 1014.5 MeV, E_K = 1109.5 MeV, 378.6 MeV, 170.9 MeV a 76.5 MeV, p = 1820 MeV/c, 923.9 MeV/c, 591.5 MeV/c a 386.5 MeV/c, λ = 0.676 fm, 1.33 fm, 2.08 fm a 3.18 fm]

14) Proton was accelerated in the electrostatic field with intensity 1 400 V/cm during time 1.5 μs, which was its kinetic energy after acceleration, how increases mass, which was its momenta, velocity, de Broglie wave length? Which intensity of magnetic field is necessary to have radius of movement of this way accelerated electron 40 cm? [E_K = 2.11 MeV, $(m-m_0)/m_0$ = 0.225 %, p = 62.95 MeV/c, v = 0.067 c, B = 0.52 T]

2) Scattering processes (kinematic, nonrelativistic, relativistic, ultrarelativistic)

1a) Proton with energy 25 MeV in laboratory system elastically scattered on deuteron in the rest to the angle 30°. Which will be energy of scattered proton (its momenta and velocity)? Which will be reflection angle of deuteron? Which will be energy, momenta and velocity of deuteron? [E_K = 21.8 MeV, p = 202 MeV/c, v = 0.21 c, E_K = 3.19 MeV, p = 109.4 MeV/c, v = 0.058 c]

1b) Neutron from fusion with energy 14 MeV in the laboratory system are elastically scattered on deuteron in the rest to the angle 130˚. Which is energy of scattered neutron (its momenta and velocity). Which are energy, momenta and velocity of deuteron? Which is recoil angle of deuteron? [$E_K(n)$ = 2,25 MeV, p = 65,0 MeV/c, v = 0,0692 c, $E_K(d)$ = 11,76 MeV, p = 210,0 MeV/c, v = 0,112 c, φ = 13,70°]

1c) Carbon scatters on proton. Which is limit of scattering angle. If carbon will have kinetic energy 12 MeV, which will be energy of reflected proton, if carbon will be scattered to angle which is half of limit angle? [ϑ_{MAX} = 4.81°, E(proton) = 0.267 MeV or 3.233 MeV]

1d) Carbon ^{12}C scatters on ^{4}He nucleus. Which part of its energy loses: a) for limit angle, b) for 0 angle c) for angle 13^0. [ϑ_{MAX} = 19.5°, a) loses 50.03 % of energy, b) loses 0 or 75.0 %, c) 8.5 % or 45.4 %]

2) Nonrelativistic deuteron scattered elastically on nucleus of carbon ^{12}C. Determine scattering angle of deuteron (m(d) = 2.013 u, m(^{12}C) = 11.997 u)

a) in laboratory coordinate system if in center of mass system is 45°
b) in center of mass system if in laboratory coordinate system is 80°
c) in center of mass system if in laboratory is 90 °

[a) ϑ = 38.9°, b) ϑ =89.51 °, c) ϑ =99.66 °]

3) Proton with energy 14 MeV was scattered on ^{235}U nucleus in the rest. Determine the nearest distance between these nuclei, if it is scattered to the angle 56˚, to the twice as much? Which is minimal kinetic energy? Which electrostatic force acts between nuclei? Use approximation with immovable uranium nucleus (infinitely heavy). [b = 8,9 fm, r_{min} =

14,8 fm, E_{kinmin} = 5,1 MeV, F_e = 0,61 MeV/fm = 97 N, b = 3,2 fm, r_{min} = 10,4 fm, E_{kinmin} = 1,3 MeV, F_e = 1,23 MeV/fm = 196 N]

4) Alpha particle with the energy 4 MeV is scattered on carbon nucleus in the rest to the angle 100°. Which is the nearest distance between nuclei? Which is kinetic energy of alpha particle during this time? Which is electrostatic force between nuclei? [r_{min} = 6,21 fm, E_{kinmin} = 0,259 MeV, F_e = 0,45 MeV/fm = 73 N]

5) Which minimal energy must have neutron to transmit by nonelastic scattering excitation energy 2.4 MeV to ^{9}Be nucleus? [Q = -2.4 MeV, E_{THRE} = 2.67 MeV]

6) Determine minimal kinetic energy of impinging alpha particle, which is needed for overcoming of Coulomb potential barrier of ^{7}Li nucleus. Will be posiible threshold reaction ^{7}Li(α,n)^{10}B for this energy? [Q = -2.790 MeV, E_{THRE} = 4.38 MeV, Coulomb barrier = 2.06 MeV]

7) For which energy of proton will be neutron produced by reaction ^{7}Li(p,n)^{7}Be in the rest in the laboratory coordinate system. [Q = -1,645 MeV, E_K = 1,921 MeV]

8) Which will be alpha decay energy of uranium 236? Which energy will have emitted alpha particle and which will be its velocity and de Broglie wave length? [Q = 4.579 MeV, E_K = 4.501 MeV, v = 0.0491 c, λ = 6.7 fm]

9) Which is reaction threshold energy of natrium 24 production on aluminum by means of neutrons? Which will be kinetic energy of produced alpha particle, if it will be emitted in the direction of neutron flight, which energy was 10 MeV. Which will be alpha particle velocity in laboratory and center of mass coordinate systems? [Q = -3.132 MeV, E_{THRE} = 3.249 MeV, $E_K(\alpha)$ = 6.70 MeV, v_α (lab) = 0,0600 c, v_α (CMS) = 0,0387 c]

10) Which is threshold energy of ^{12}C(n,d)^{11}B reaction. Which is energy of deuteron scattered to the angle 20 degrees, if projectile neutron has energy two times value of threshold energy? [Q = -13,732 MeV, E_{THRE} = 14,887 MeV, E_{KIN_D} = 15,7025 MeV]

11) To which energy is necessary to accelerate proton to produce proton antiproton pair during collision with proton in the fixed target? Which energy is needed, if proton collides with nucleon in target nucleus which is moving against it by Fermi motion and nucleon kinetic energy is 25 MeV. [E_K = 5628 MeV (6 rest masses of proton), 4292 MeV (4.58 rest masses of proton)]

12) Particle Ξ^0 (mass 1314.8 MeV/c^2) decays to hyperon Λ (m = 1115.7 MeV) and π^0 meson (mass 135,0 MeV). Determine kinetic energy and momenta of hyperon and meson after decay. [Q = 64.1 MeV, E_K(hyperon) = 8.1 MeV, E_K (meson) = 56.0 MeV, p = 135.0 MeV/c]

13) Which is threshold energy of tauon production (m = 1776.8 MeV/c^2) by collision of tauon neutrino with proton in the rest? [E_{THRE} = 3459.6 MeV]

14) Velocity of hot and dense zone, which is produced by heavy ion collision is in laboratory coordinate system v = 0,85c. Velocity of this zone in the center of mass system is 0 (also rapidity is zero). Determine velocity of proton, which is emitted from this zone to the angle ϑ = 0° or 180° in laboratory system, if center of mass system has velocity v = 0.94c. Calculate by means

of rapidity. Prove by relativistic summation of velocities. [$y=y_1+y_2$= 1.2562+1.7380=2.9942, v = 0.9950 c, $y=y_1-y_2$=-0.4818, v = -0.45 c]

15) Range of rapidity from -1.5 to 1.5 is necessary to study in the center of mass system during experiment. Which range of rapidity is necessary to cover In which range of rapidity we need to measure in laboratory system? Center of mass system moves against laboratory system with velocity 0.932c. Which is appropriate interval of velocities? [rapidities from 0,17 up to 3.17, velocities from 0.168c up to 0.9965c]

16) Prove, that sum of Mandelstam variables is equal to sum of squares of rest masses of all participated particles.

3) Cross-section

1) Aluminium (A_r = 26.98, ρ = 2700 kg/m^3) has cross-section of thermal neutron absorption 0,23 barn. Which is absorption length of thermal neutrons in the aluminium? To which value decreases neutron intensity in 0.6 mm, 8 mm, 80 mm and 1 m aluminium? [λ_{abs} = 0,721 m, intensity of neutrons will decrease to 99,92 %, 98,89 %, 89,49 % and 24,98 %]

2) Natural coper (ρ = 8930 kg/m^3) consists of two isotopes ^{63}Cu (69.1 %) a ^{65}Cu (30.9 %). Their absorption cross-sections are 4.5 barn a 2.2 barn. Which will be absorption length of thermal neutrons in coper, which will be macroscopic cross-section? Which will be decreasing of thermal neutrons in layer of copper with thickness 0.5 mm, 2 mm a 10 mm. [λ = 0.0313 m, Σ = 31.9 m^{-1}, decrease will be 1.58 %, 6.19 % a 27.3 %.

3) Which will be maximal probability of neutrino interaction with given energy during flight through Earth, if cross-section of neutrino interaction with nucleon is $\sigma_{\nu N} = 10^{-14}$ barn, mean Earth density ρ = 5500 kg · m^{-3} and their radius R_Z = 6378 km. [0.0042 %]

4) Total cross-section of 1 GeV proton reaction with aluminum foil (thickness 10 mm) is 0.8 barn and cross-section of ^{24}Na production is 15times smaller. Which is mean free path of this proton? Which part of beam is absorbed and which part produces sodium 24? Density of aluminum is 2700 kg/m^3, A_r = 26.98. [λ = 0.207 m, 4.71 % of beam is absorbed and 0.31 % produces ^{24}Na]

5) Bismuth foil activation detector has thickness 0.95 mm and form of square with size of 1 cm. Activity induced during irradiations was homogenously distributed in the sample. Which part of gamma photons irradiated to the direction of detector are absorbed by self-absorption? Which are absorption lengths in bismuth for gammas with energies 100 keV and 200 keV? Total cross-sections of gamma with these energies in bismuth are 1999 barns and 359 barns. Density of bismuth is 9780 kg/m^3 and its Ar = 208.98. [Absorption length is 0.178 mm and 0.991 mm, 81.4 % and 35.7 % of gamma photons are absorbed].

4) Properties of nuclei (sizes, masses, …)

1) Determine radius and energies of lead and hydrogen muon atom shells by means of Bohr atom model. [E(Pb) = 18,91 MeV·(1/n^2), E(H) = 2,52 keV·(1/n^2) with taken into account movement of hydrogen nucleus, R(Pb) = 3,12 n^2]

2) Circumference of storage ring ESR at GSI Darmstadt is 108 m. Determine, which must be magnetic induction to store ^{198}Au nuclei with momentum 1280 A·MeV/c. Assume that magnets a distributed equally and circumference will be circular by (reality is different). Which will be difference between velocities and frequencies of ^{198}Au nucleus and isomeric state ^{198m}Au with energy 811.7 keV. Momenta of both nuclei were accurately equalized by electron cooling. [B=0,62 T, $\Delta f/f=\Delta v/v=1{,}52\cdot10^{-6}$, Δv=369 m/s, Δf=3,41 Hz]

3) Tandetron used for mass spectrometry accelerates once charged ions to energy 4 MeV. Which will be velocity of ^{12}C ions with this energy, which will be energy of ^{13}C and ^{14}C ions with the same velocity? Which will be ratio between intensities of electric and magnetic field for velocity filter for such velocity? Which will be intensity of magnetic field of spectrometer to have radius of once charged ion of ^{14}C will be 60 cm? Determine differences between radiuses of different carbon isotopes. [E/B = 7,44 $MVm^{-1}T^{-1}$, B = 1,79 T]

5) Nuclear models (droplet, fermi gas, shell ...)

1) Determine by means of Weizsäker formulae the most stable isobar with nucleon number 188. [Z = 77 – agrese with real situation]

2) Determine difference of Fermi energy for protons and neutrons for ^{238}U nucleus by means of Fermi model? Determine mean kinetic energy of protons and neutrons? [$E_F(n)$=38,13 MeV, $E_F(p)$=28,09 MeV, $E_{KIN}(n)$=22,88 MeV a $E_{KIN}(p)$=16,85 MeV]

3) Determine masses of ^{60}Co, ^{197}Au and ^{238}U nuclei by means of Weizsäker formulae. Compare them with real values. Determine energy of alpha decay for ^{238}U nucleus. Compare it with real value. [Weizsäcker: $E_0(^{60}Co)$ = 55,8172 GeV, $E_0(^{197}Au)$ = 183,4331 GeV, $E_0(^{238}U)$ = 221,6850 GeV, Real situation: $E_0(^{60}Co)$ = 55,8145 GeV, $E_0(^{197}Au)$ = 183,4328 GeV, $E_0(^{238}U)$ = 221,6958 GeV, Differences in the order of MeV are done by nuclei asymmetry or nearness of magic numbers, Energy of ^{238}U is not possible to calculate]

6) Radioactive decay (decay law, different decays …)

1) Product of ^{238}U decay is ^{226}Ra. Determine half-time of ^{238}U decay, if on one ^{226}Ra nucleus we have $2{,}8\cdot10^{6}$ of ^{238}U nuclei. Half-time of ^{226}Ra decay is 1620 years. [$T_{1/2}(^{238}U)$ = 4,5 miliard let]

2) Determine mass of lead produced from 1 kg of ^{238}U during time comparable with age of Earth ($2.5\cdot10^{9}$ years). Half-time of ^{238}U is $4{,}46\cdot10^{9}$ years.

3) Velocities of different radionuclides activity increasing during irradiation of gold foil were: ^{196}Au 190 Bq/s, ^{194}Au 300 Bq/s a 193Au pak 120 Bq/s. Gold foil was nonradioactive before irradiation. Which will be activities of particular radionuclides after 15 hours of irradiation and 5 hours of cooling? Half-times of particular radionuclides are 6.183 d, 38.02 h and 17.65 h.

4) Iodine 135 (half-time 6.58 hours) is often produced by fission and high amount of this radionuclide is produced in thermal nuclear reactors. Xenon 135 (half-time 9.17 h) is produced in the fission with smaller probability and it has very high neutron absorption cross-section. High amount of iodine 135 and negligible amount of xenon 135 are in reactor just after its switching off. Xenon 135 is produced by iodine 135 decay. Determine ratio between numbers

of xenon and iodine 135 after 12 hours from reactor switching off? Which will be ratio of activities of xenon and iodine?

5) Isomeric state of yttrium 87 is produced by reaction (n,3n) from yttrium 89. It decays with half-time 13.38 hours by gamma decay to ground state of yttrium 87. The ground state of yttrium 87 with decay half-time 79.8 hours is also produced. The activity 355 Bq of isomeric state of yttrium 87 and activity 21 Bq of ground state are produced every minute during irradiation of yttrium foil by neutrons. Determine activity of ground and isomeric states after 20 hours of irradiation.