Joint seminar of the NPI of the CAS

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Vladimir Lotoreichik, Ph.D., DSc. (DTP NPI): Asymptotic analysis of Dirac operators on thin domains and with small inclusions

Abstract:

In this talk, we will discuss two problems involving asymptotic analysis of Dirac operators with small parameters describing the geometry of the system. As the small parameter tends to zero, the spectral properties in both problems are governed by an effective operator. We will first consider the Dirac operator with infinite mass boundary conditions on a tubular neighbourhood of a smooth compact hypersurface in Rn. In three dimensions, these boundary conditions correspond to the famous MIT bag model. We will discuss the asymptotic behaviour of the eigenvalues of this Dirac operator when the tubular neighbourhood shrinks to the hypersurface. It turns out that this asymptotic behaviour is driven by a Schrödinger operator on the hypersurface involving electric and Yang-Mills potentials of geometric nature. We will also address the homogenization of the twodimensional Dirac operator with position-dependent mass. The mass is piecewise constant and supported on small pairwise disjoint inclusions evenly distributed along an ε -periodic square lattice. Under rather general assumptions on geometry of these inclusions we prove that the corresponding family of Dirac operators converges as $\varepsilon \rightarrow 0$ in the norm resolvent sense to the Dirac operator with a constant effective mass provided the masses in the inclusions are adjusted to the scaling of the geometry.