AAMP XXI meeting

including the special session on spectral theory

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→ Book of Abstracts ←

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Magnetic quantum graphs with time-reversal non-invariant vertex coupling

Marzieh Baradaran

Abstract: Motivated by the application of quantum graphs to model the anomalous Hall effect, we discuss spectral properties of magnetic quantum graphs assuming that the vertex coupling is of a particular type violating the time-reversal invariance. We show how these two mechanisms, the magnetic field, and the vertex condition, compete. Special attention is paid to the asymptotic behavior of the spectral bands in the high-energy regime. We see that the Band-Berkolaiko universality holds as long as the graph edges are incommensurate. The talk is based on joint works with Pavel Exner and Jiri Lipovsky [1-3].

References:

[1] M. Baradaran and P. Exner, Cairo lattice with time-reversal non-invariant vertex couplings, J. Phys. A: Math. Theor. 57, (2024), 265202.

[2] M. Baradaran, P. Exner and J. Lipovsky, Magnetic square lattice with vertex coupling of a preferred orientation, Ann. Phys. 454, (2023), 169339.

[3] M. Baradaran, P. Exner and J. Lipovsky, Magnetic ring chains with vertex coupling of a preferred orientation, J. Phys. A: Math. Theor. 55, (2022),375203.

Polynomial algebras in the universal enveloping algebras of simple exceptional Lie algebras

Rutwig Campoamor-Stursberg

Abstract: During the last years, there has been a wide interest to find purely algebraic formulations of superintegrable systems, by associating them to polynomial algebras of various types, such as subalgebras of enveloping algebras, Racah algebras and generalizations [1,2] . In this context, large series of polynomial algebras related to the classical semisimple series have been studied [3], trying to find characterizations that lead to new superintegrable systems. For the exceptional Lie algebras, this analysis is still missing, mainly due to computational obstructions. In this talk, we provide an algorithmic approach to the study of certain commutants of the exceptional algebras, which allows to determine the generic structure of the polynomial algebras associated to a Cartan subalgebra.

This work is part of a collaboration with D. Latini, D., I. Marquette, Y.-Z. Zhang and J. Zhang.

References:

[1] Létourneau, P. and Vinet, L. *Superintegrable systems: polynomial algebras and quasi-exactly solvable Hamiltonians*, Annals Phys. **244** (1995), 144–168.

[2] Campoamor-Stursberg, R. and Latini, D. and Marquette, I. and Zhang, Y.-Z., *Polynomial algebras from Lie algebra reduction chains* $\mathfrak{g} \supset \mathfrak{g}'$, Annals Phys. **459** (2023), 169496.

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Transport and topological properties in phosphorus chains: A Review on tight-binding models

Pedro Domínguez Rodríguez

Abstract: Tight-binding models are used to study the energy band structure and electronic properties of periodic systems. These type of models allow us to highlight different transport and topological features of those type of systems and have proven successful in determine the properties of many real and novel materials, such as the polyacetylene molecule, graphenic materials, heterostructures, transition metal dichalcogenides and low dimensional materials such as phosphorene nanoribbons and borophene. They also allow for quick computational calculations and to be implemented experimentally in classical equivalente systems such as topoelectrical circuits, photonic crystals and mechanical resonators. In this talk, I analyze the topological features of a polymeric red phosphorus chain and present a review of other results obtained using tight-binding models done in solid state research.

$n-$particle representations of some $Sp(2k,R)$ groups using the orthogonal group $O(n)$

Hubert de Guise

Abstract: We present a simple algorithm for the explicit construction of $n-$particle harmonic oscillator states simultaneously belonging to irreducible representations of $Sp(2,R)$ (or $SU(1,1)$) and $O(n)$. The construction can be done using generating functions or hyperspherical harmonics, with the cases of $n=2$ and $n=3$ investigated at greater length. The similar problem of constructing states for $Sp(4,R)$ irreps and eventually $Sp(6,R)$ will also be discussed, with emphasis on $O(n)$ tensors methods, highlighting the differences with $Sp(2,R)$.

In collaboration with Noah Kakekaspan (McGill University, Montreal)

Flow from generic complex non-hermitian matrices to normal matrices and the fate of the Single Ring Theorem.

Joshua Feinberg

Abstract: Unitary invariant probability ensembles of NxN generic complex non-hermitian matrices are subject to the Single Ring Theorem in the large-N limit. That is to say, the support of the average density of eigenvalues in the complex plane in this limit has the shape either of a disk or an annulus. This is so independently of the profile of the density of singular values of the corresponding ensemble. In contrast, unitary invariant random non-hermitian normal matrix models are not subjected to the Single Ring Theorem. In the large-N limit, their eigenvalues may condense in concentric rings, which are determined essentially by the segments along the positive real axis where the density of singular values is supported.

In this talk I will discuss a random matrix model which flows from generic complex to normal matrices as a parameter increases. I will show that a universal feature of this flow is change in the statistics of singular values from Wigner-Dyson type repulsion to non-repellent Poisson statistics. I will also present numerical results which demonstrate the cross over from single-ring to multi-ring behavior of the density of eigenvalues in the complex plane.

This work was done in collaboration with Roman Riser, Richard Scalettar and Tony Zee.

A Conditionally Exactly Solvable 1D Dirac Pseudoscalar Interaction Potential

Astghik Ghazaryan

Abstract: We study an analytically solvable pseudoscalar interaction potential for the one-dimensional stationary Dirac equation, which consists of power terms proportional to x^{-1}, $x^{-1/3}$, and $x^{1/3}$. This potential is classified as conditionally exactly solvable due to the fixed strength of the first term at a specific constant. We present the general solution to the Dirac equation in terms of non-integer index Hermite functions, which are distinct from the conventional integer index Hermite polynomials. We analyze the energy spectrum of the bound states and the eigenfunctions and compare the results with the case without the $x^{-1/3}$ term.

References:

1. Atkins, P.W., Quanta: A handbook of concepts, Oxford: Oxford University Press, 1991.

2. A. M. Ishkhanyan, V.P. Krainov, Conditionally exactly solvable Dirac potential, including $x^{1/3}$ pseudoscalar interaction Physica Scripta 98, 2023.

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Exact Solution of Modified Lance-Emden equation and its application

Hassan Hassanabadi

Abstract: In this work, we consider a deviation for the gravity and based on we obtain the modified Lane-Emden equation by solving this problem we determined the exact solution of the deformed Lane-Emden equation and based on we calculated the internal energy and pressure, and also calculated the radius of the white dwarf with the pressure due to degeneration and gravity.

In collaboration with Won Sang Chung (Gyeongsang National University), Jan Kriz (Univ. of Hradec Králové), Fariba Kafikang (Shahrood University of Technology)

Complete Classification of Local Conservation Laws for Generalized Cahn–Hilliard–Kuramoto–Sivashinsky Equation

Pavel Holba

Abstract: In this talk we consider the following PDE in $n+1$ independent variables $t, x_1 \dots, x_n$ and one dependent variable u,

$$u_t = a\Delta^2 u + b(u)\Delta u + f(u)|\nabla u|^2 + g(u), \tag{1}$$

to which we refer as to the generalized Cahn–Hilliard–Kuramoto–Sivashinsky equation, as it is a natural generalization of nonlinear multidimensional Cahn–Hilliard and Kuramoto–Sivashinsky equations that have many important applications in physics, chemistry, and biology. Here b, f, g are arbitrary smooth functions of the dependent variable u, a is a nonzero constant, n is a natural number, $\Delta = \sum_{i=1}^{n} \partial^2/\partial x_i^2$ is the Laplace operator and $|\nabla u|^2 = \sum_{i=1}^{n} (\partial u/\partial x_i)^2$.

For an arbitrary natural number n of spatial independent variables we present a complete list of cases when equation (1) admits nontrivial local conservation laws of any order, and for each of those cases we give an explicit form of all the local conservation laws of all orders modulo trivial ones admitted by the equation under study.

As a consequence of the above general result, we show that the Kuramoto–Sivashinsky equation,

$$u_t = -(\Delta^2 u + \Delta u + |\nabla u|^2/2),$$

admits no nontrivial local conservation laws, and list all of the inequivalent nontrivial local conservation laws for the Cahn–Hilliard equation

$$u_t = c_1 \Delta(u^3 - u + c_2 \Delta u),$$

where c_1 and c_2 are arbitrary nonzero constants.

References

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Shadows of new physics on analog systems, GUPs and other amusements

Alfredo Iorio

Abstract: After a brief introduction to the analog enterprise in general, and to Dirac materials in particular, I shall show how, when higher-order effects in the parameter $\ell/\hbar|p|$, related to the lattice spacing ℓ, are considered, Dirac materials can be used as tabletop systems where generalized commutation relations are naturally realized. Such algebras, which lead to generalized versions of the Heisenberg uncertainty principle, are under intense scrutiny these days, as they could be the effect of a fundamental length scale of space. Despite the efforts and the many intriguing theoretical results, though, there are no experimental signatures of any generalized uncertainty principle (GUP). Therefore, our results here could be of interest to practitioners of the phenomenology of quantum gravity. We identify three different energy regimes that we call "layers", where the physics is still of a Dirac type, but within precisely described limits. With the goals just illustrated, we had to identify the mapping between the high-energy coordinates, X_i, and the low-energy variable, x_i, p_i, i.e., those measured in the lab. We obtain four natural generalized Heisenberg algebras, three for commuting X_is, including $X_i = x_i$, and one for noncommuting X_is. This last instance we found quite noticeable. I shall conclude by discussing a recipe for seeing all that in laboratories.

Klein-Gordon potentials solvable in terms of the general Heun functions

Artur Ishkhanyan

Abstract: We study the potentials for which the one-dimensional stationary Klein-Gordon equation can be solved in terms of the general Heun function, a special function of the new generation that generalizes the ordinary hypergeometric function. We reveal that, under the assumption of independent variability of all potential parameters, there are only 35 possible forms of the independent variable transformation that allow reduction of the Klein-Gordon equation to the general Heun equation, each of these transformations generating an unconditionally exactly solvable potential. Due to the symmetry of the Heun equation with respect to the transposition of its singularities, only 11 out of 35 admissible potentials are independent; the others can be derived from these 11 by specifying the involved parameters to particular (real or complex) values. Since one of these latter potentials is a constant, there are just 10 nontrivial independent potentials. Four of these potentials are six-parametric; the other six are five-parametric. Four of the potentials possess ordinary hypergeometric sub-potentials. We present explicit solutions of the Klein-Gordon equation using the general Heun functions for these 10 independent potentials, and provide a comprehensive representation of the involved parameters.

Atomic Bound States at Critical Coupling

Michal Jex

Abstract: N-electron atom undergoes unbinding for a critical charge of the nucleus Z_c, i.e. the atom has discrete eigenstates for the case $Z > Z_c$ and it has no bound states for $Z < Z_c$. The question what happens at the critical coupling is subtle. In the talk we present how to obtain asymptotic decay rate of the ground state at critical coupling under the assumption that $Z_c < N - K$, where K is the number of electrons removed from the atom to be stable for $Z = Z_c$. In this setting the decay rate satisfies

$$\psi(x) \lesssim \exp\left(- \sum_{j=N-K}^{N} \sqrt{\Delta E_j} |x|_j - \sum_{k=1}^{K} 2(N-k-Z_c)\sqrt{|x|_k} \right),$$

where ΔE_j corresponds to ionization energies and $|x|_k$ is the distance of k-th outer most electron to the nucleus. Furthermore we show how our method improves estimates on the decay rate in subcritical setting.

Towards an algebraic approach to gravitational quantum mechanics

Georg Junker

Abstract: Most approaches towards a quantum theory of gravitation indicate the existence of a minimal length scale of the order of the Planck length. Quantum mechanical models incorporating such an intrinsic length scale call for a deformation of Heisenbergs algebra resulting in a generalized uncertainty principle and constitute what is called gravitational quantum mechanics. Using an explicit position representation, the free time evolution of a Gaussian wave packet is investigated as well as the spectral properties of a particle bound by an external attractive potential. Here the cases of a box with finite width and infinite walls, and an attractive potential well of finite depth are considered.

Joint work with Won Sang Chung and Hassan Hassanabadi.

On the Ginzburg-Landau Energy of Corners

Ayman Kachmar

Abstract: It is a well known fact that the geometry of a superconducting sample influences the distribution of the surface superconductivity for strong applied magnetic fields. For instance, the presence of corners induces geometric terms described through effective models in sector-like regions. We study the connection between two effective models for the offset of superconductivity and for surface superconductivity introduced by Fournais–Bonnaillie-Noël and Correggi-Giacomelli, respectively. We prove that the transition between the two models is continuous with respect to the magnetic field strength, and, as a byproduct, we deduce the existence of a minimizer at the threshold for both effective problems. Furthermore, as a consequence, we disprove a conjecture stated by Correggi-Giacomelli.

Quantum Counterpart of Energy Equipartition Theorem

Jasleen Kaur

Abstract: In classical statistical mechanics, for a system at thermal equilibrium, the thermally-averaged kinetic energy is given by the equipartition theorem, wherein the result is $kBT/2$ per degree of freedom, independent of the potential forces acting between the degrees of freedom and the strength of coupling between the system and the heat bath (which maintains thermal equilibrium). This result changes dramatically for quantum systems, and more so when the system and the heat bath are coupled strongly. In my talk, I will discuss the formulation of the quantum counterpart of the energy equipartition theorem for a quantum oscillator interacting with a heat bath, with the bath itself modeled as a collection of infinitely-many independent quantum oscillators. The key ingredient towards this mathematical formulation is a fluctuation-dissipation theorem due to Callen and Welton, and which appears in the linear-response formalism of the Brownian motion problem. I will show that the mean kinetic energy can be expressed as a two-fold average; the first averaging is performed over the Gibbs canonical state of the heat bath, while the second averaging is performed over a suitable probability distribution defined to be a positive-definite and normalized function of the frequencies of the heat bath. This is an example of superstatistics which involves the superposition of two statistics. The different control parameters, as in the harmonic-trap eigenfrequency, the system-bath coupling strength, and the memory timescale for heat-bath excitations are found to influence the kinetic energy of the quantum oscillator. The expression for the kinetic energy can be recasted in the form of an infinite series over the bosonic Matsubara modes and simliar discussion follows for the potential energy. Finally, I will also briefly discuss the generalization of these results for a charged oscillator in a magnetic field which is a dissipative version of the Landau problem, relevant to diamagnetism and Hall effect.

In collaboration with Aritra Ghosh and Malay Bandyopadhyay (Indian Institute of Technology Bhubaneswar, India)

Homogenization of the Dirac operator with position-dependent mass

Andrii Khrabustovskyi

Abstract: We address the homogenization of the two-dimensional Dirac operator with position-dependent mass. The mass is piecewise constant and supported on small pairwise disjoint inclusions evenly distributed along a periodic square lattice. Under rather general assumptions on geometry of these inclusions we prove that the corresponding family of Dirac operators converges in the norm resolvent sense as the lattice period goes to zero to the Dirac operator with a constant effective mass provided the masses in the inclusions are adjusted to the scaling of the geometry. We also estimate the speed of this convergence in terms of the scaling rates. This is a joint work with V. Lotoreichik (NPI CAS).

Symplectic-Haantjes geometry of first order magnetic (super)integrable systems

Ondřej Kubů

Abstract: Despite significant effort in the new millennium, (super)integrable systems immersed in magnetic fields are not well understood, mainly because the 1:1 correspondence with separation of variables of Hamilton-Jacobi (HJ) equation is broken. Moreover, the standard theory of separation of variables on the configuration space requires to fix the gauge in a somewhat ad hoc manner. In this talk, we report on the first application of the symplectic-Haantjes formalism to (super)integrable magnetic systems. The algorithm for diagonalizing the Haantjes operators introduces coordinates in which the Hamiltonian attains an explicitly separable form regardless of the choice of gauge. We uncover the generalized Stäckel form of the algebra, and obtain, apparently for the first time, some families of integrable systems with magnetic fields on Riemannian spaces with nonconstant curvature.

Based on O. Kubů et al., Hamiltonian integrable systems in a magnetic field and symplectic-Haantjes geometry. *Proc. R. Soc. A* (2024) in press [arXiv:2401.16897].

Systematic classification of solvable potentials within and outside the Natanzon class

Géza Lévai

Abstract: A standard way to generate exactly solvable potentials is the transformation of the Schrödinger equation into the second-order differential equation of some special function of mathematical physics. When this special function is the (confluent) hypergeometric function, then one obtains the Natanzon (confluent) potential class, which contains the most well-known solvable potentials of quantum mechanics. Their bound states are typically expressed in terms of classical ortogonal polynomials (Jacobi, generalized Laguerre, etc.). Recently we reformulated this transformation method and obtained a set of algebraic equations for each special function that connects the energy eigenvalue, the principal quantum number n, the parameters of the special function, the coupling coefficients of the potential and the parameters appearing in the variable transformation function. For the Natanzon (confluent) potentials this set consists of three algebraic equations that have to be solved simultaneously in order obtain an exactly solvable potential. This scheme also serves for the basis of classifying the solvable potentials in terms of the special function and the variable transformation function (1). More recently we extended this formalism to various versions of the Heun equation. In this case the set consists of five of algebraic equations. We considered the confluent Heun equation (2), including its symmetrical canonical form that is more suitable to discuss PT-symmetric potentials (3). The $X1$ type exceptional Laguerre polynomials were also recovered as the special polynomial solutions of the confluent Heun equation (2). The differential equation of the $X1$ type exceptional Jacobi polynomials has also been discussed (4). The analysis of the remaining versions of the Heun equation, such as the bi-confluent one (5), within this scheme is underway.

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The magnetic Laplacian on the Disc for strong constant magnetic fields

Germán Miranda

Abstract: The study of the Neumann realization of the magnetic Laplacian traces back to works of Saint-James and de Gennes in the context of superconductivity. On a planar domain and under a strong magnetic field, the magnetic Laplacian has eigenvalues close to the Landau levels. When the domain is a disc, the spectrum consists of branches of eigenvalues of one dimensional operators.

Using a variational approach, we derive asymptotics of the eigenvalues of the Neumann realization of the magnetic Laplacian under a strong constant magnetic field with accurate estimates of exponentially small remainders. Our approach allows us to recover recent results by Baur and Weidl for the Dirichlet realization.

This is joint work with Ayman Kachmar.

References:

[1] A. Kachmar, G. Miranda, "The magnetic Laplacian on the Disc for strong magnetic fields" preprint arXiv:2407.11241 (2024)

Dynamical Formulation of Stationary Scattering, Renormalization of Point Scatterers, and Exactness of N-th-order Born Approximation

Ali Mostafazadeh

Abstract: In two and three dimensions, the standard treatment of the scattering problem for a multi-delta-function potential, $v(x) = \sum_{n=1}^{N} z_n \delta(\mathbf{r} - \mathbf{a}_n)$, leads to divergent terms. Regularization of these terms and renormalization of the coupling constants z_n give rise to a finite expression for the scattering amplitude, but this expression has an important short-coming; in the limit where the centers $\mathbf{a}_n$ of the delta functions coincide, it does not reproduce the formula for the scattering amplitude of a single-delta-function potential, i.e., it seems to have a wrong coincidence limit. We show that the application of a recently developed dynamical formulation of stationary scattering (DFSS) provides a finite expression for the scattering amplitude with correct coincidence limit. Another remarkable outcome of DFSS is the discovery of a class of complex potentials whose scattering problem is exactly solvable by the N-th order Born approximation, where N depends on the wavenumber of the incident wave.

References:

- F. Loran and A. Mostafazadeh, Renormalization of multi-delta-function point scatterers in two and three dimensions, the coincidence-limit problem, and its resolution, Ann. Phys. (NY) **443**, 168966 (2022); arXiv: 2204.09554.

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- F. Loran and A. Mostafazadeh, Can N-th order Born approximation be exact?, J. Phys. A: Math. Theor. **57**, 335205 (2024); arXiv: 2407.1993.

Some recent applications of the Generalized Uncertainty Principle

Luis Miguel Nieto

Abstract: Due to the importance of a unified theory of quantum mechanics and gravity, and the existence of a minimal length (Planck scale), we will analyze two problems:

A) A modified Schrödinger equation resulting from a generalized uncertainty principle (GUP), with a quantum mechanically corrected gravitational interaction recently proposed. The resulting equation cannot be solved by common exact approaches and therefore we use a Bethe ansatz approach.

B) Dirac equation is considered with a generalized gravitational interaction which includes post-Newtonian (relativistic) and quantum corrections to the classical potential. The spacetime contributions are contained in an external potential or in an electromagnetic potential. We also discuss several known generalizations of the Coulomb potential within this formulation in terms of certain Heun functions. The Bethe-ansatz approach is proposed to overcome this challenging problem.

References:

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Generalization of the Coleman-Mandula theorem and ring paradigm

Jan Novák

Abstract: We introduce a new approach to the quantization of gravity called the ring paradigm, according to which graviton is a phonon on a dynamical grid. The only way to conceptually verify this paradigm is to show that there exists a certain generalization of the Coleman-Mandula theorem. We review the basic no-go theorems in QFT and we show that the proofs of these theorems rely heavily on the use of mathematical tools that are peculiar to at space. It is not immediately obvious how to generalize it to other situations. Further, for the ring paradigm it will be important that the Lorentz group is a non-compact group. And we need to ask what type of theorems we will obtain for the case of compact groups.

Estimates for Periodic and Antiperiodic Eigenvalues of the Schrödinger operator with the Kronig-Penney Model

Cemile Nur

Abstract: We give estimates for small periodic and antiperiodic eigenvalues of the one-dimensional Schrödinger operator $H(q)$, for a special real, periodic, and locally integrable potential q in the case of the Kronig-Penney model. We obtain some useful equations derived from some iteration formulas for calculating periodic and antiperiodic eigenvalues using Rouche's theorem. Then, to estimate the eigenvalues numerically, we take finite sums instead of the infinite series in the resulting equations and use numerical methods. Moreover, we give error estimations and present a numerical example.

Key Words: Eigenvalue estimations, Periodic and antiperiodic boundary conditions, Kronig-Penney model.

Non-standard quantum algebras and finite dimensional PT -symmetric systems

Marta Reboiro

Abstract: In this work, PT -symmetric Hamiltonians defined on quantum $sl(2,R)$ algebras are presented. We study the spectrum of a family of non- Hermitian Hamiltonians written in terms of the generators of the non-standard $U_z(sl(2,R))$ Hopf algebra deformation of $sl(2,R)$. By making use of a particular boson representation of the generators of $U_z(sl(2,R))$, both the co-product and the commutation relations of the quantum algebra are shown to be invariant under the PT -transformation. In terms of these operators, we construct several finite dimensional PT -symmetry Hamiltonians, whose spectrum is analytically obtained for any arbitrary dimension. In particular, we show the appearance of Exceptional Points in the space of model parameters and we discuss the behaviour of the spectrum both in the exact PT -symmetry and the broken PT - symmetry dynamical phases. As an application, we show that this non-standard quantum algebra can be used to define an effective model Hamiltonian describing accurately the experimental spectra of three-electron hybrid qubits based on asymmetric double quantum dots. Remarkably enough, in this effective model, the deformation parameter z has to be identified with the detuning parameter of the system.

In collaboration with Ángel Ballesteros (Universidad de Burgos), and Romina Ramírez (University of La Plata)

$\mathcal{PT}$-symmetric oscillators with one-center point interactions

Iveta Semorádová

Abstract: We investigate the spectrum of Schrödinger operators with imaginary polynomial potentials in $L^2(R)$, perturbed with δ, or δ' interaction, centered at the origin

$$-\partial_x^2 + ix^{2k-1} + \alpha\delta, \quad -\partial_x^2 + ix^{2k-1} + \beta\delta', \tag{2}$$

where $\alpha \in R$, $\beta \in R$, $k \in N$.

It is well established that the spectrum of the unperturbed operators consists of countable many real, isolated and simple eigenvalues for $k \geq 2$, and it is empty for $k = 1$.

When $\alpha \neq 0$ or $\beta \neq 0$, for $k \geq 1$, we observe countable many non-real eigenvalues appearing in complex conjugate pairs, and at maximum finitely many real eigenvalues. The non-real eigenvalues asymptotically converge to the eigenvalues of the unperturbed problems defined on $L^2(R_+)$ and $L^2(R_-)$ with Dirichlet, resp. with Neumann boundary conditions for δ, resp. δ' interaction.

Moreover, for $\alpha \leq C_k < 0$, we show the existence of negative real eigenvalue, diverging to $-\infty$ as $\alpha \to -\infty$.

References

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[2] M. Marletta, I. Semorádová, $\mathcal{PT}$-symmetric oscillators with one-center point interactions *manuscript in preparation*

Integrable systems of the ellipsoidal, paraboloidal and conical type with magnetic field

Libor Šnobl

Abstract: We construct integrable Hamiltonian systems with magnetic fields of the ellipsoidal, paraboloidal and conical type, i.e. systems that generalize natural Hamiltonians separating in the respective coordinate systems to include nonvanishing magnetic field. In the ellipsoidal and paraboloidal case each this classification results in three one-parameter families of systems, each involving one arbitrary function of a single variable and a parameter specifying the strength of the magnetic field of the given fully determined form. In the conical case the results are more involved, there are two one-parameter families like in the other cases and one class which is less restrictive and so far resists full classification.

Multi-step method to solve the Schrödinger equation

Raul Valencia

Abstract: We consider position-dependent effective mass particles in double heterostructures, subject to the action of several potentials. A comparative study of the energy spectra of these particles is carried out, emphasizing the effect of different boundary conditions associated with the kinetic energy operator. Energy spectra that cannot be expressed analytically, have been estimated by means of the corresponding reflection coefficient poles. The heterostructure model adopted assumes a middle region, where the potential and mass are some finite distributions of the position, but outside of it its behavior is constant. Finally, we discuss the gain of a double parabolic quantum well in a laser.

Conservation laws and nonexistence of local Hamiltonian structures for generalized Infeld–Rowlands equation

Jakub Vašíček

Abstract: We exhaustively characterize all cases when a certain natural generalization of the Infeld–Rowlands equation admits nontrivial local conservation laws of any order, and give explicit form of these conservation laws modulo trivial ones. The original Infeld–Rowlands equation arises inter alia in the study of the stability of the Ginzburg–Landau equation. What is more, one of the special cases of our generalization can be seen as an extension of the two-dimensional Kuramoto–Sivashinsky equation to dimension three.

It turns out that even in the generic case the equation in question, to which we refer to as the generalized Infeld–Rowlands equation, admits an infinite family of nontrivial local conservation laws parameterized by an arbitrary smooth function of one variable.

Furthermore, we prove that the equation under study admits no nontrivial local Hamiltonian structures and no nontrivial local symplectic structures, no nontrivial local Noether and no nontrivial local inverse Noether operators no matter the orders of the structures or operators in question; the method of establishing the said nonexistence results can be readily applied to many other PDEs.

For further details please see J. Vašíček, Conservation laws and nonexistence of local Hamiltonian structures for generalized Infeld –Rowlands equation, Rep. Math. Phys. 93 (2024),287-300.

On differential operators with PT-symmetric coefficients having a purely real spectrum

Oktay Veliev

Abstract: We consider the Bloch eigenvalues and the spectrum of a non-self-adjoint differential operator L generated by a differential expression of odd order n with the periodic PT-symmetric coefficients, where $n > 1$. The localizations of the Bloch eigenvalues and the structure of the spectrum are studied. In addition, conditions were found for the norm of the coefficients under which the spectrum of the operator L is purely real and coincides with the real line.

The bound state of a weakly coupled Schrödinger operator with a complex potential in one and two dimensions

Nicolas Weber

Abstract: We consider the (not necessarily self-adjoint) Schrödinger operator of the form $H_\beta = -\Delta - V_\beta$ in $L^2(R^d)$, $d5 \in \{1, 2\}$, where $V_\beta : \mathbb{R}^d \to \mathbb{C}$ is a complex- valued potential, depending on some parameter $\beta \in \mathbb{C}$, such that $V_\beta \to 0$ as $\beta \to 0$ in an appropriate sense. We derive sufficient conditions, depending on the decay behaviour of V_β and the spatial dimension, for the existence of an eigenvalue $\lambda_\beta \in \sigma_p(H_\beta)\backslash[0, \infty)$, as $\beta \to 0$, and also obtain an asymptotic expansion for this eigenvalue. Finally, we provide conditions on V_β, under which H_β never has an eigenvalue in $\mathbb{C}\backslash[0, \infty)$, as $\beta \to 0$.

Inverse Feshbachs effective Hamiltonian problem

Miloslav Znojil

Abstract: The description of dynamics of open systems is often based on the Feshbach's concept of model space (projector P). Mathematically this means that the full-space Hamiltonian H of a system in question is replaced by its energy-dependent model-space avatar

$$H_{eff}(E) = P\,H\,P + P\,H\,Q\,\frac{Q}{E - Q\,H\,Q}\,Q\,H\,P$$

where $Q = I - P$. In the talk the inverse problem will be considered. The effective Hamiltonian will be assumed known (as, typically, fitting the experimental data) while the full-space Hamiltonian H (describing also the open-system environment) will be reconstructed.