Shadows of new physics on analog systems, GUPs and other amusements

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Analytic & algebraic methods in physics XXI
Czech Technical University
Prague

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Talk based on

A.I.,

Boris Ivetić, 

 *Salvatore Mignemi,

Pablo Pais, 

*PRD 106 (2022) 116011 [2208.02237]

PLB 853 (2024) 138630 [2306.17196]

See also (ancestor)

IJMPD 27(2018)1850080 [1706.01332]

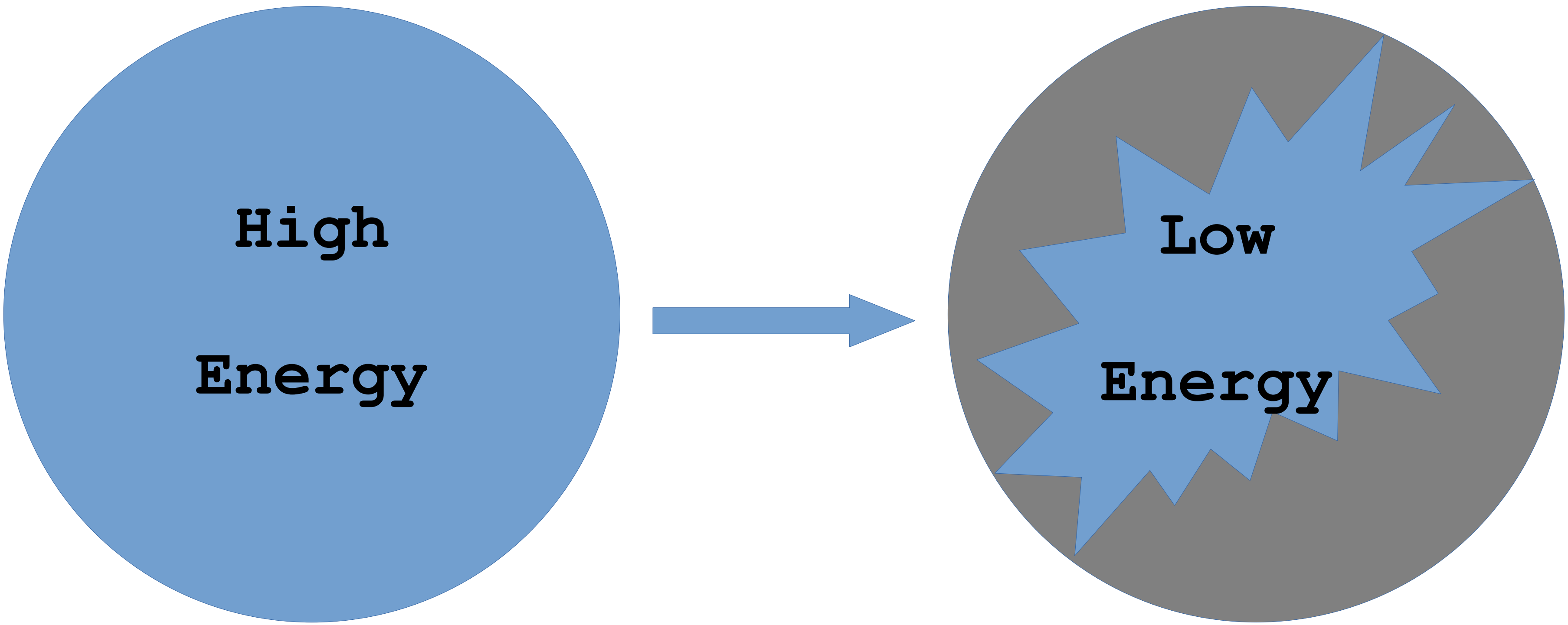
and (general framework)

G.Acquaviva, A.I., P.Pais, L.Smaldone,
Hunting quantum gravity with analogs: the case of graphene, Universe 8(2022)455 [2207.04097]

• Shadows of new physics

It is an established tradition to look for effects of high energy phenomena at our low energy scales

$$\mathcal{S}(\hat{\Phi}_a) = \mathcal{S}(\Phi_a) + S_1(\epsilon, \Phi_a, \partial) + O(\epsilon^2, \partial^2)$$



$$\mathcal{S}(\hat{\Phi}_a) \qquad \mathcal{S}(\Phi_a) + S_1(\epsilon, \Phi_a, \partial) + O(\epsilon^2, \partial^2)$$

E.g., noncommutative field theories (Seiberg-Witten map)

$$\hat{A}_\mu = A_\mu - \frac{1}{2}\theta^{\alpha\beta}A_\alpha(\partial_\beta A_\mu + F_{\beta\mu}) + O(\theta^2)$$

$$-\frac{1}{4}\hat{F}\cdot\hat{F} = -\frac{1}{4}\left(F\cdot F - \frac{1}{2}(\theta\cdot F)\,(F\cdot F) + 2(F\theta F)\cdot F\right) + O(\theta^2)$$

$$\theta \sim \ell_P^2$$

Lorentz violation (Colladay-Kostelecky's SM Extension)

$$\mathcal{S}(\hat{\Phi}_a) = \mathcal{S}_{SM}(\Phi_a) + \sum_{k=1}^{\infty} C^{(k)}_{\mu\ldots\nu}(\Phi_a, \partial)^{\mu\ldots\nu}$$

$$C^{(k)} \sim \ell_P^k$$

These are the days of GUPs. E.g., high energy (x, p) vs low energy (x_0, p_0)

$$p_i = p_{0i} \left(1 - A|\vec{p_0}| + 2A^2|\vec{p_0}|^2\right)$$

where $A = \tilde{A}\,\ell_P/\hbar$

$$x_i = x_{0i}$$

with

$$[x_{0i}, p_{0j}] = \imath\hbar\delta_{ij}$$

$$[x_i, p_j] = \imath\hbar\left[\delta_{ij} - A|\vec{p}|\left(\delta_{ij} + \frac{p_i p_j}{|\vec{p}|^2}\right) + A^2|\vec{p}|^2\left(\delta_{ij} + 3\,\frac{p_i p_j}{|\vec{p}|^2}\right)\right]$$

S. Das and E. C. Vagenas, Phys. Rev. Lett. **101**, 221301 (2008)

Intense activity to find GUP-corrected physics

E.g., the GUP-corrected Dirac equation (any dimension)

$$\begin{aligned} H\,\psi &= \left(c\vec{\alpha}\cdot\vec{p} + \beta mc^2\right)\,\psi \\ &= \left(c\vec{\alpha}\cdot\vec{p}_0 - cA(\vec{\alpha}\cdot\vec{p}_0)^2 + \beta mc^2\right)\,\psi \\ &= E\,\psi \end{aligned}$$

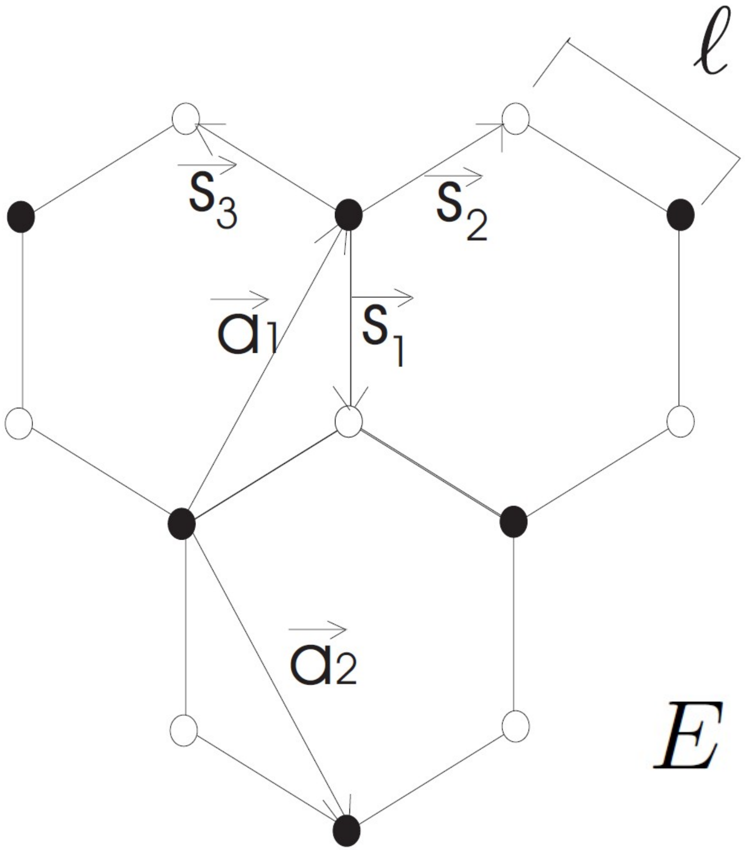
that gives (two dimensions)

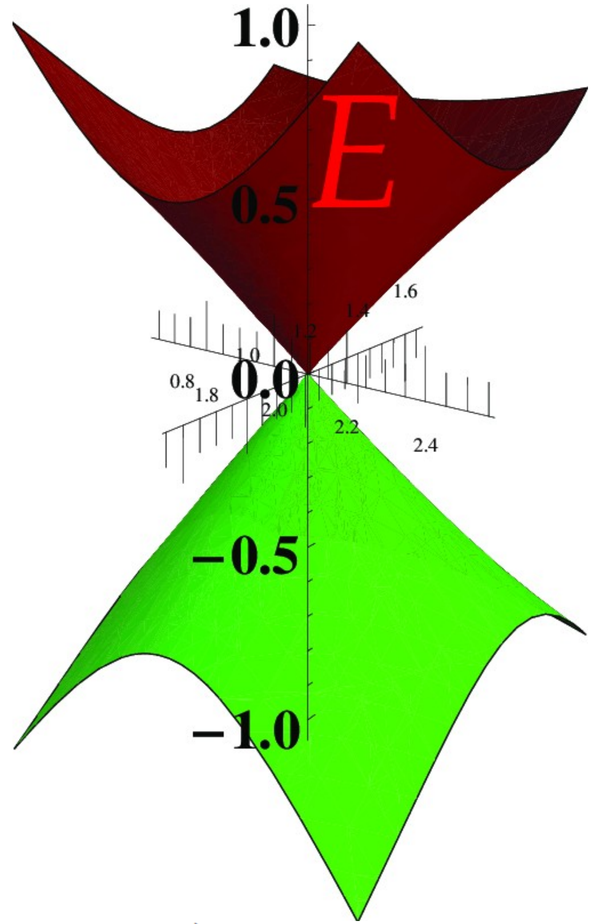
$$\left(-\imath\hbar c\sigma_1 \frac{d}{dx_0} + A\,\hbar^2 c\,\frac{d^2}{dx_0^2} + \beta mc^2\right)\,\psi = E\,\psi$$

S. Das, E. C. Vagenas, and A. F. Ali, Phys. Lett. **B690**, 407 (2010)

All of this is fascinating, but... what does it mean? and, can we put our hands on it?

- **Graphene, the newcomer**





$$E = \pm v_F |\vec{p}|$$

$$H \;=\; v_F \sum_{\vec{p}} \psi^\dagger_{\vec{p}} (\sigma_1 \, p_1 \;+\; \sigma_2 \, p_2)\, \psi_{\vec{p}}$$

with $v_F = 3/2\, \eta\, \ell / \hbar \simeq 0.003\, c$

Two-dimensional gas of massless Dirac fermions in graphene

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M
& A. A. Firsov[2]

PHYSICAL REVIEW D **90**, 025006 (2014)

Q ... theory in curved graphene spacetimes, Lobachevsky ...mmetry, Hawking effect, and all that

V Holešovičkách 2,

Chiral tunnelling and the Klein paradox in graphene

M. I. KATSNELSON[1]*, K. S. NOVOSELOV[2] AND A. K. GEIM[2]*

[1]Institute for Molecules and Materials, Radboud University Nijmegen, 6525 ED Nijmegen, The Netherlands
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The Hawking–Unruh phenomenon on graphene

Alfredo Iorio [a,*], Gaetano Lambiase [b,c]

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A. K. Geim & K. S. Novoselov

Nature Materials 6, 183–191 (2007) | Cite this article

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...CAL REVIEW B **105**, L161401 (2022)

Letter

Effects of discrete topology on quantum transport across a graphene *n*-*p*-*n* junction: A quantum gravity analog

To have intrinsic curvature, $\mathcal{K}$, in an (hexagonal) lattice, disclination defects are necessary

$$\sum_p (6-p)n_p = 6\chi_M \qquad (\clubsuit)$$

and

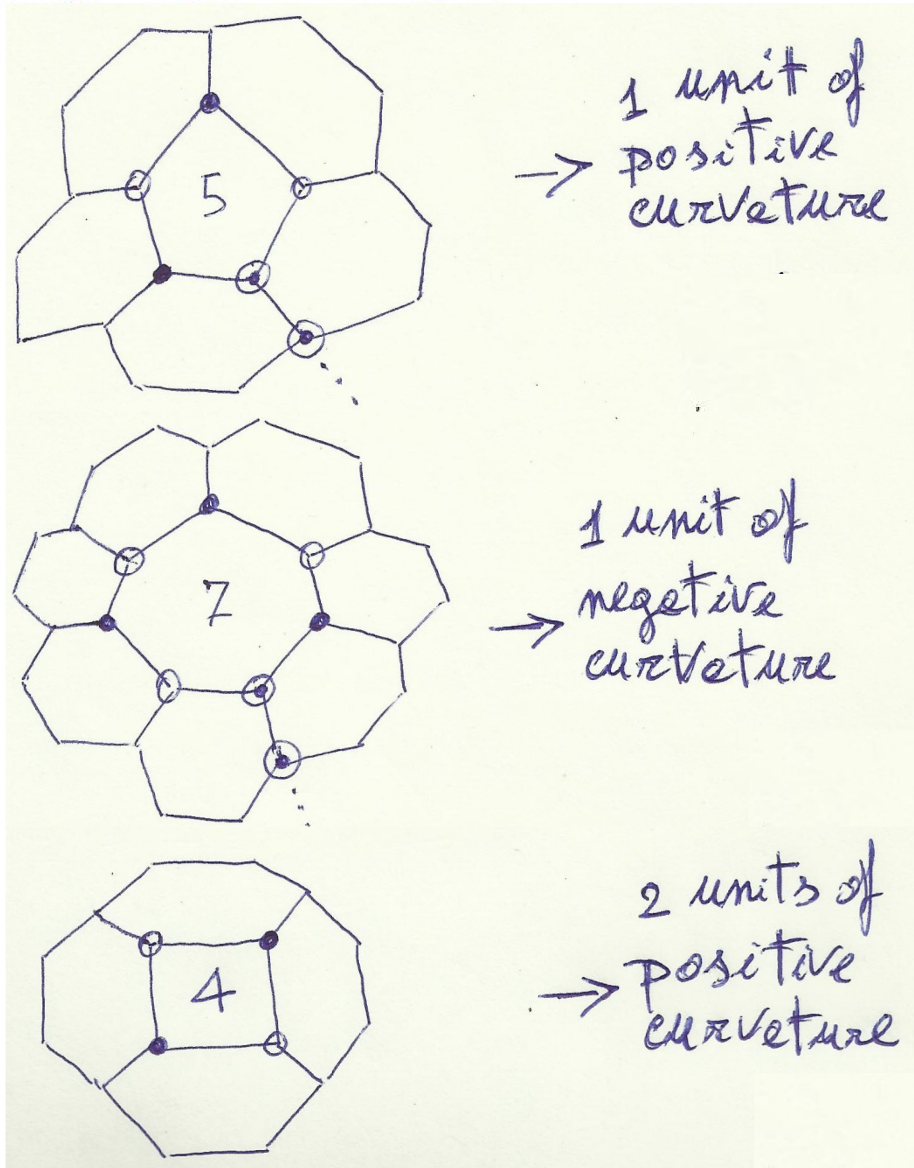
$$\int_M \mathcal{K}(x) \equiv \mathcal{K}_{tot} = 2\pi\chi_M \qquad (\spadesuit)$$

E.g., $M = S^2$ $(\chi_{S^2} = 2)$

$$(6-7)\,n_7 + (6-6)\,n_6 + (6-5)\,n_5 = 12$$

that is: n_6 **irrelevant,** $n_5 = 12 + m$, $n_7 = m$

Thus, (♣) and (♠) together give



$$\mathcal{K}_5 = +\left(\frac{3}{\pi}\right)\frac{\mathcal{K}_{tot}}{12}$$

and

$$\mathcal{K}_7 = -\left(\frac{3}{\pi}\right)\frac{\mathcal{K}_{tot}}{12}$$

and so on

This is behind Ω_μ

This is *exotic* for graphene, but *meagre* for hep-th

Local Weyl symmetry, for $n = 3$, comes to our rescue

When

$$g_{\mu\nu} \to \phi^2 g_{\mu\nu}$$

and

$$\psi \to \phi^{-1} \psi$$

the classical action

$$\mathcal{A} \to \mathcal{A}$$

Particularly important are the cases of conformal flatness

$$g_{\mu\nu} = \phi^2 \eta_{\mu\nu}$$

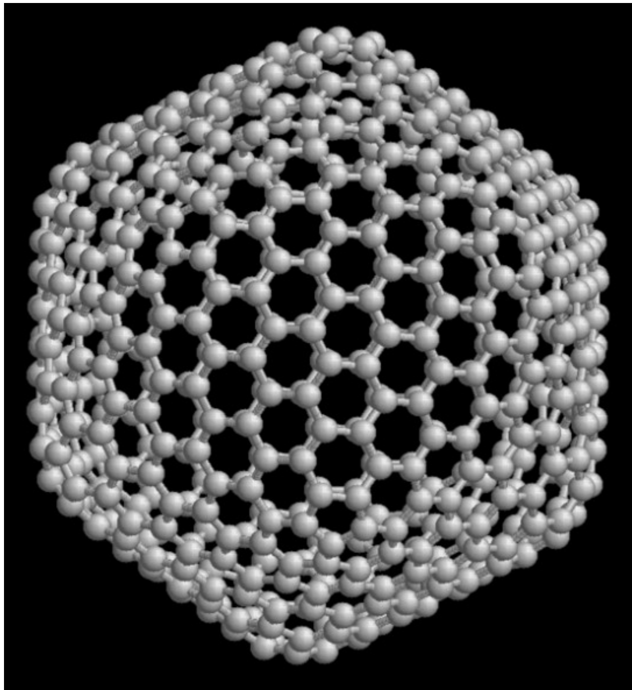
How can we make CF *spacetimes* with

$$g^{2+1}_{\mu\nu}(x,y) = \begin{pmatrix} 1 & 0 \\ 0 & g^{(2)}_{\alpha\beta}(x,y) \end{pmatrix}$$

The condition is

$$C_{\mu\nu} = \epsilon_{\mu\lambda\kappa}\nabla^{\lambda} {R^{(3)}}^{\kappa}_{\ \nu} + \epsilon_{\nu\lambda\kappa}\nabla^{\lambda} {R^{(3)}}^{\kappa}_{\ \mu} = 0$$

All surfaces of constant Gaussian curvature $\mathcal{K}$, give a CF spacetime!



One immediately thinks of the sphere

$$\mathcal{K} = \frac{1}{r^2}$$

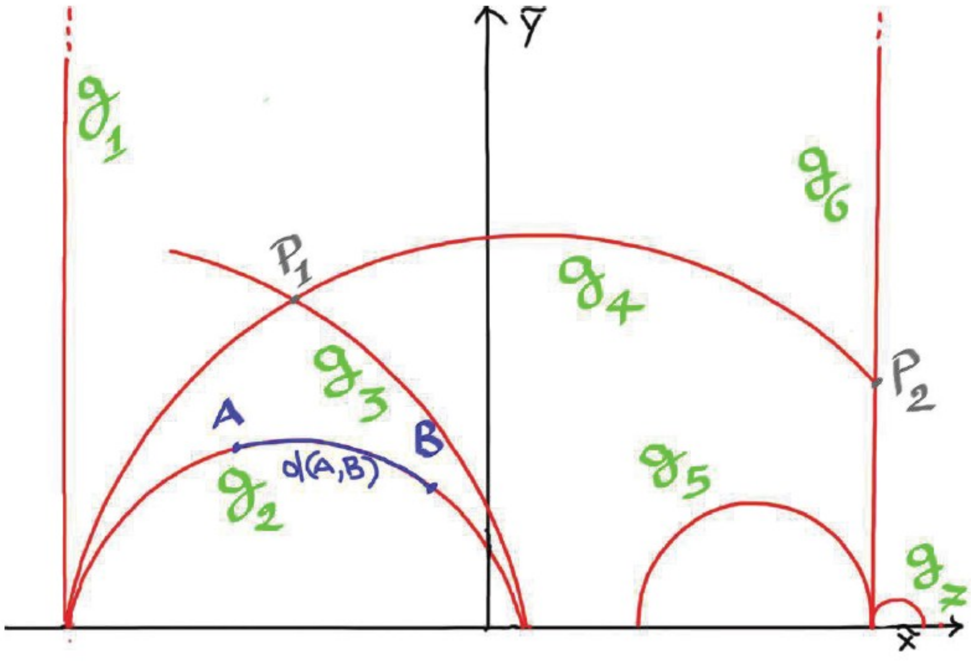
Interesting, but no horizons in sight...

There is another case, that is

$$\mathcal{K} = -\frac{1}{r^2}$$

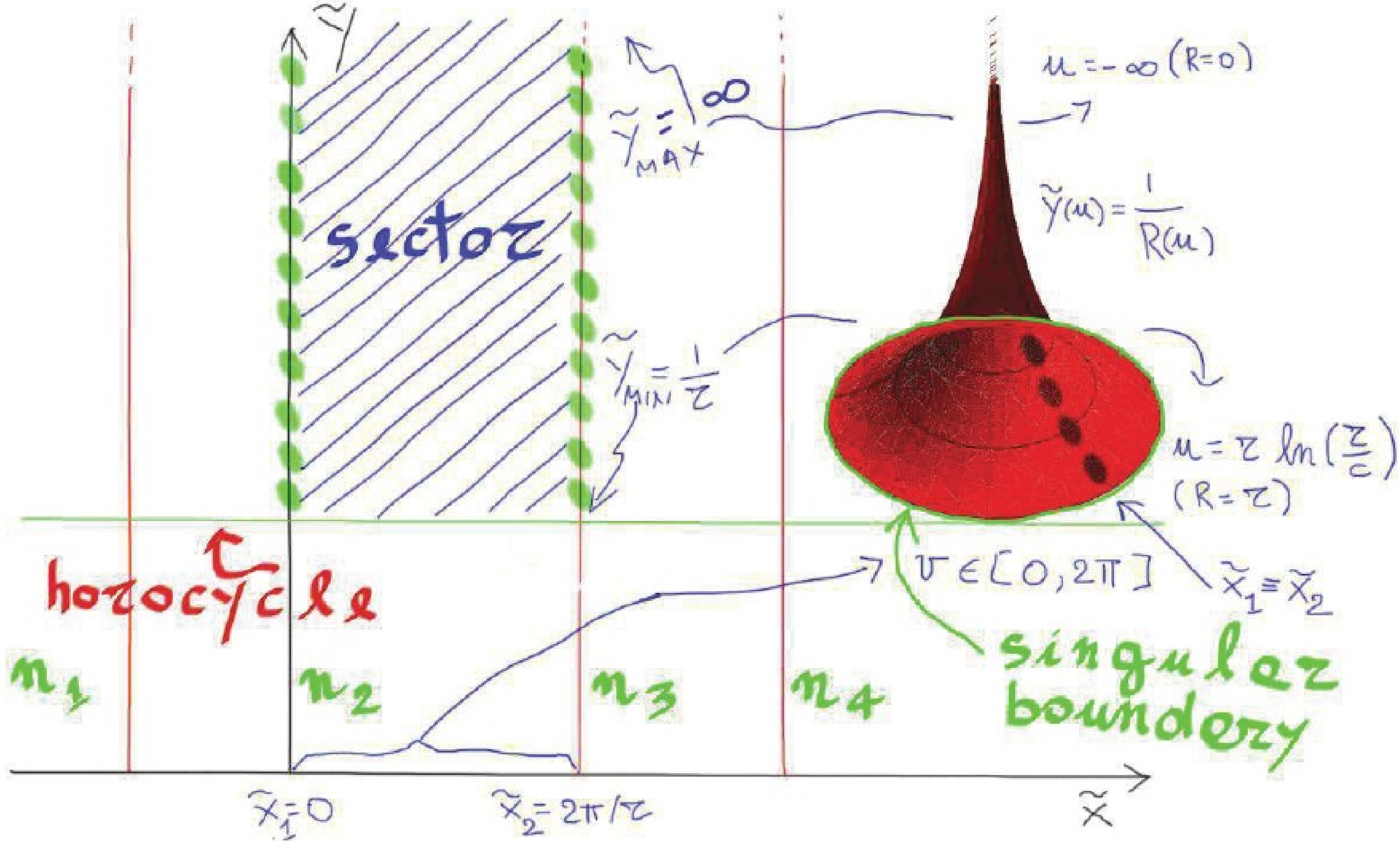
which brings us into Lobachevsky geometry

$$ds^2_{\text{graphene}} = dt^2 - \frac{r^2}{\tilde{y}^2}(d\tilde{x}^2 + d\tilde{y}^2)$$

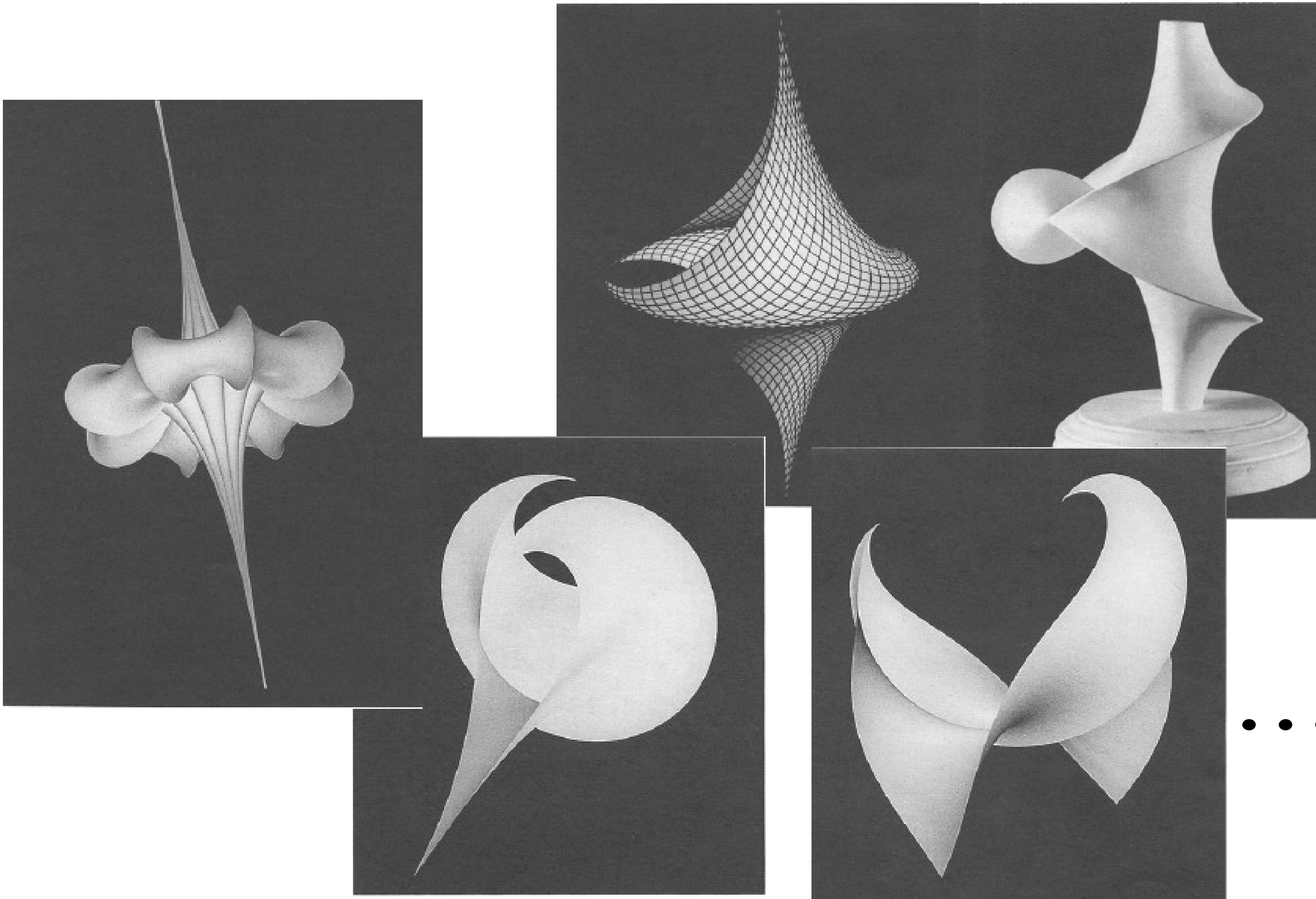


PRD 90 (2014) 025006

$$\tilde{x} = \frac{v}{r} \quad \tilde{y} = \frac{e^{-u/r}}{c} \qquad v \in [0, 2\pi] \quad u \in [-\infty, r\ln(r/c)]$$



(Infinitely) many directions to explore



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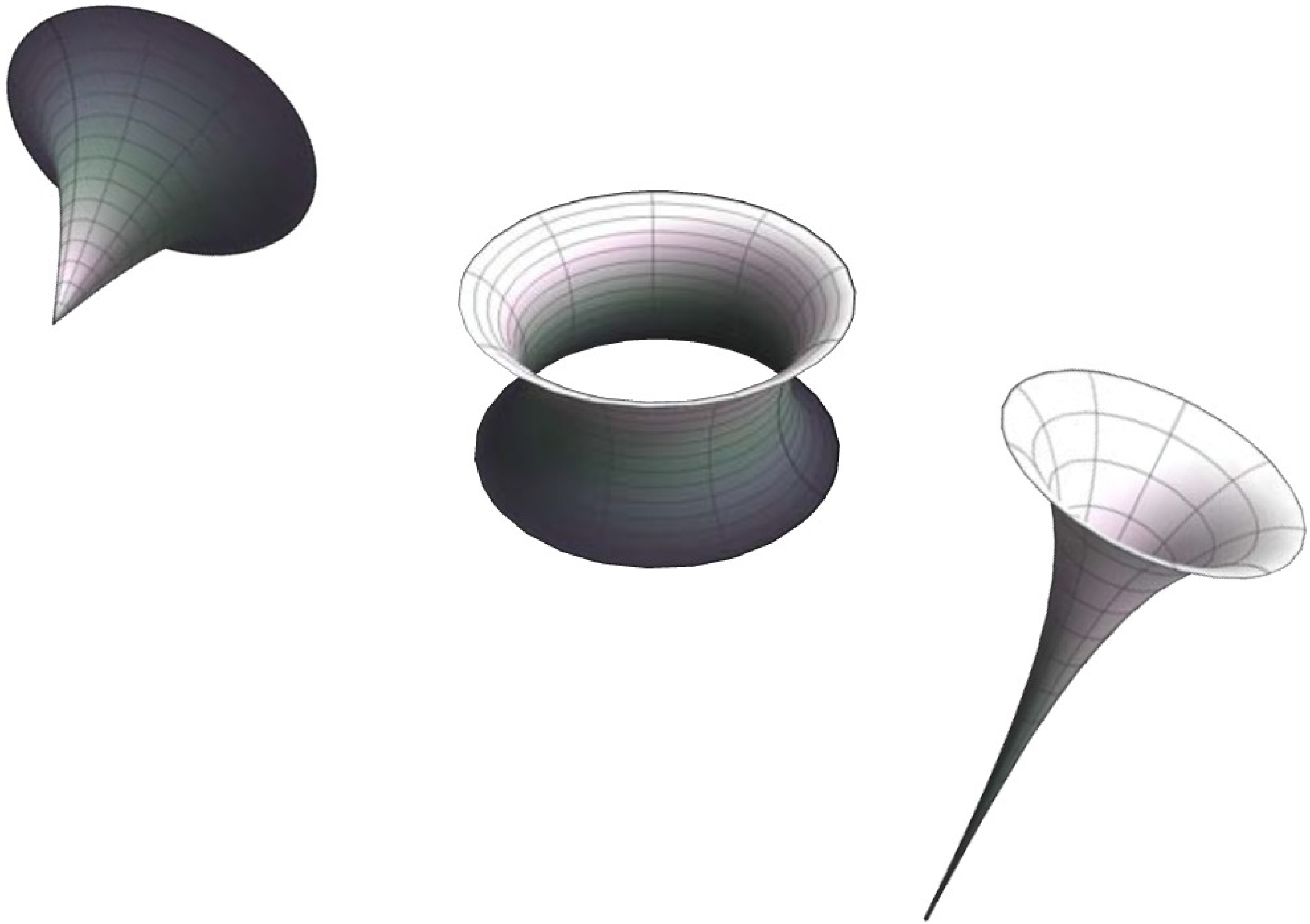
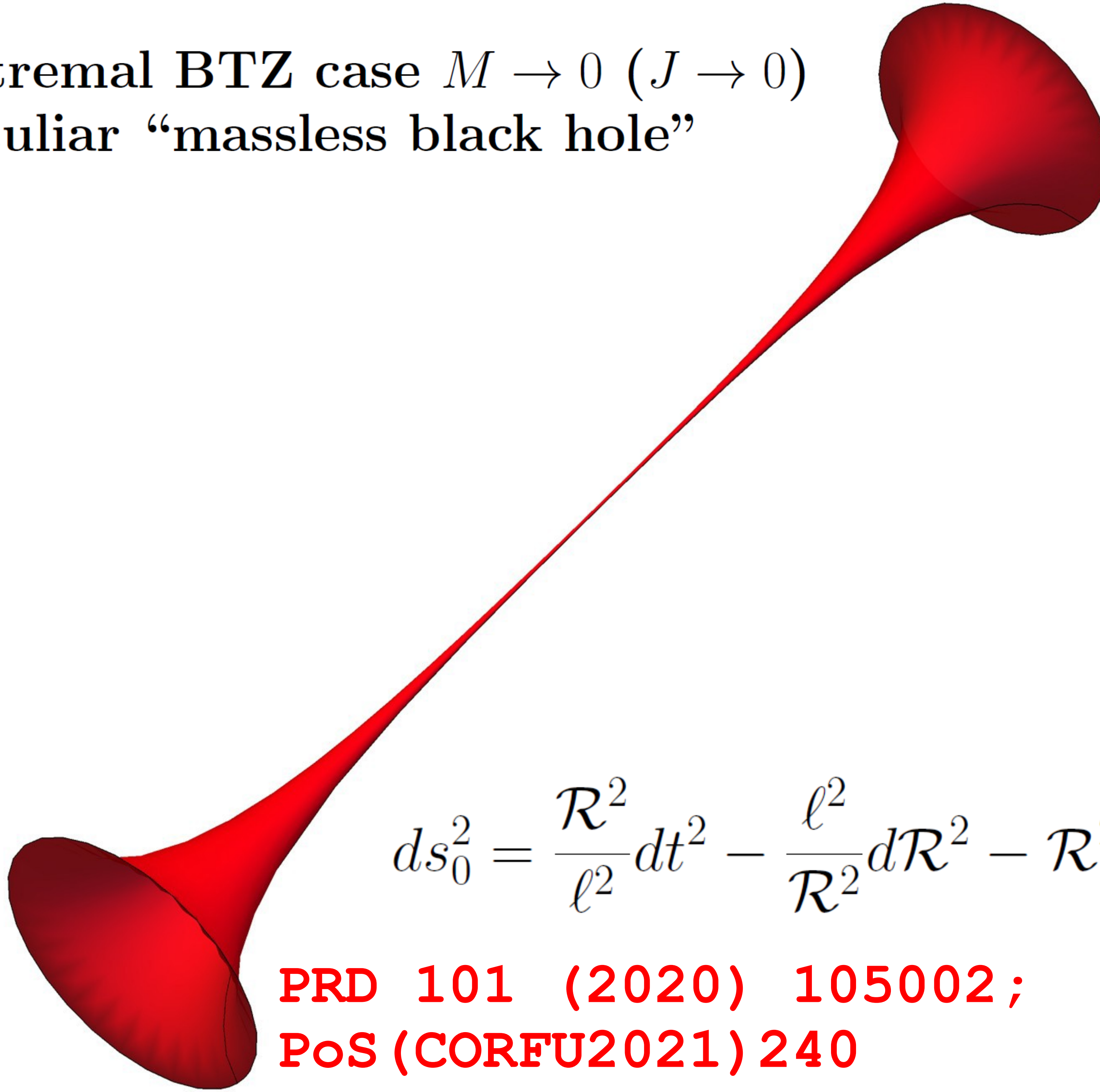
PRD 90 (2014) 025006

R McLachlan, Maths Int 16 (1994) 31



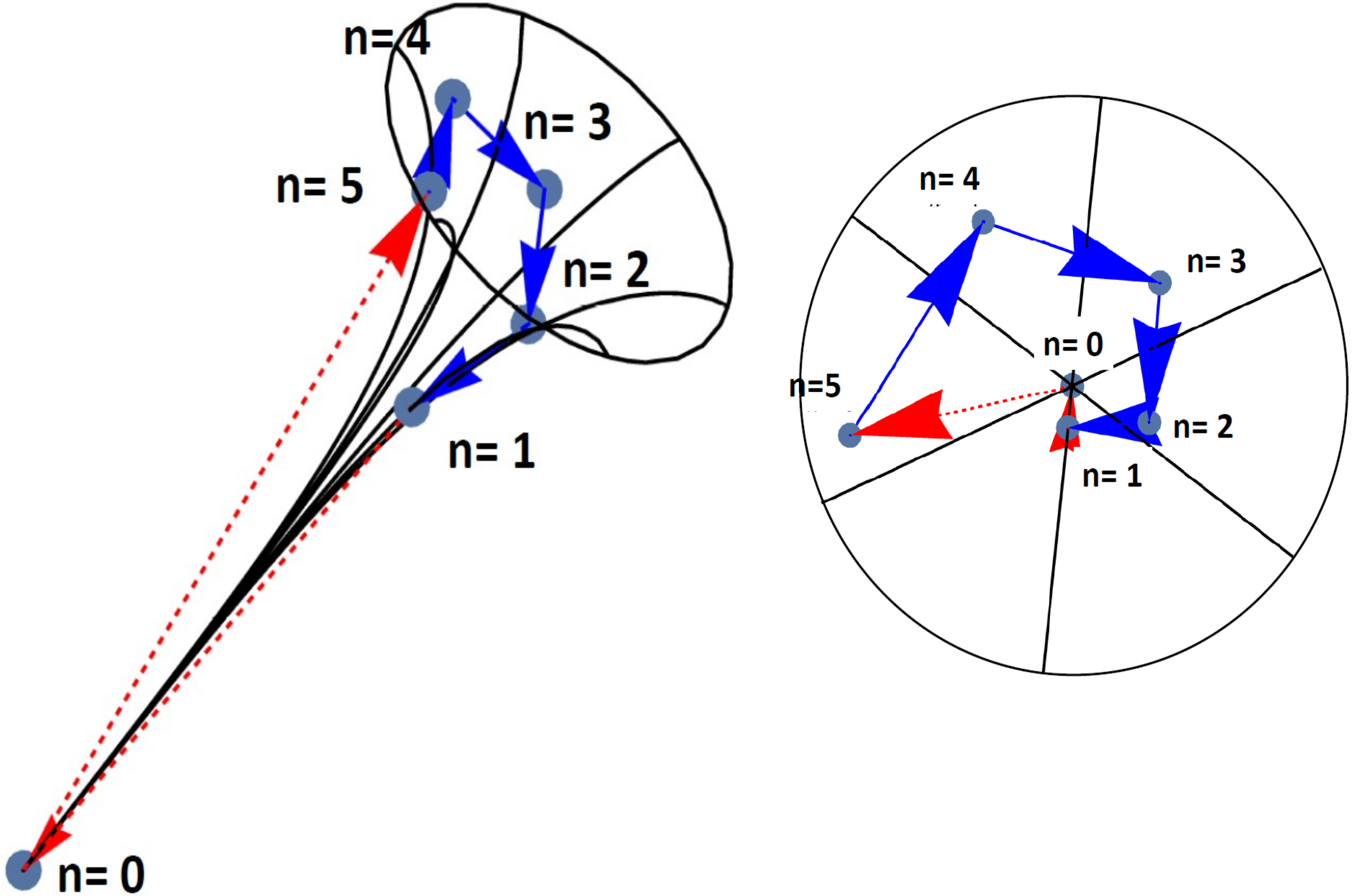
The extremal BTZ case $M \to 0$ ($J \to 0$) is a peculiar "massless black hole"



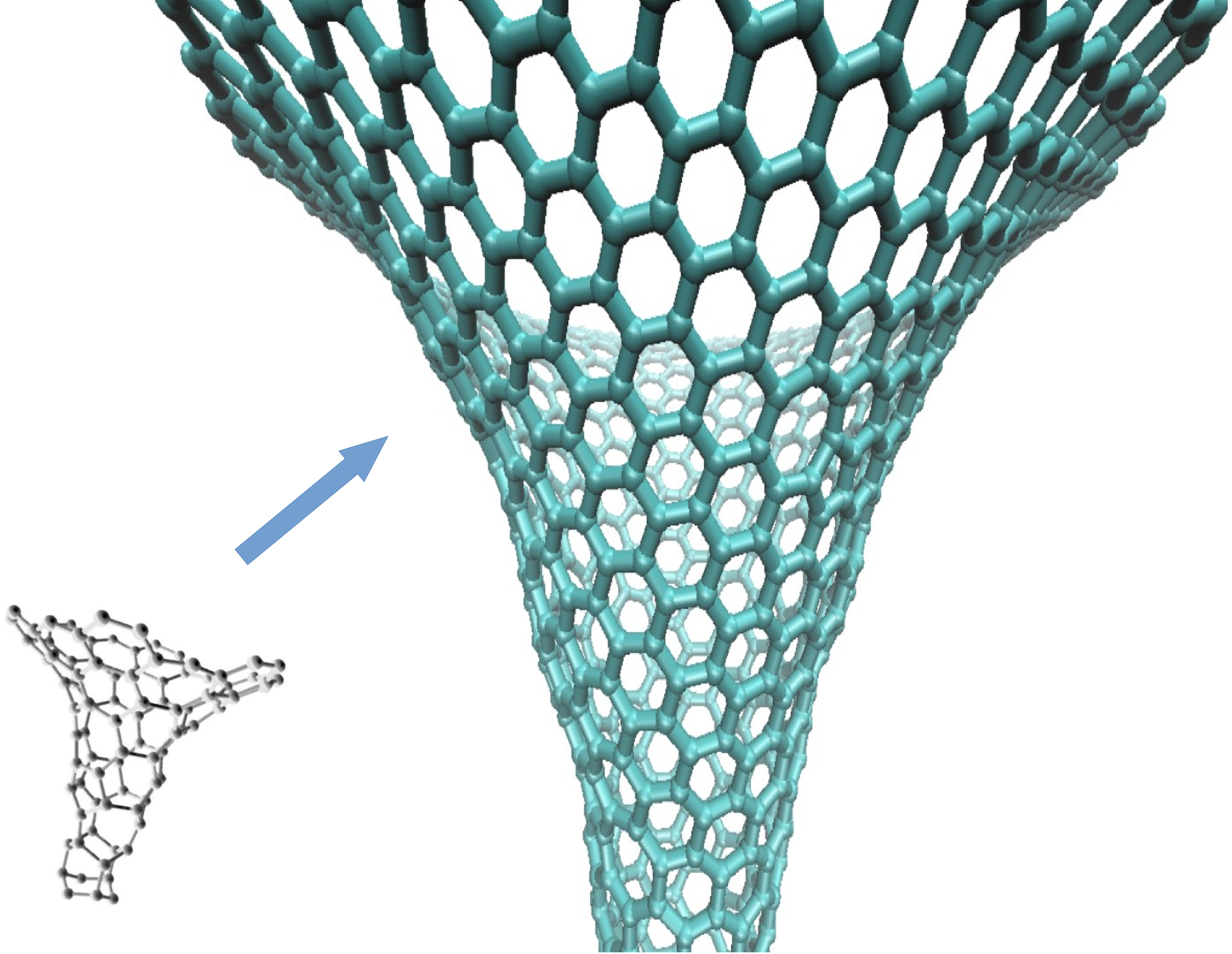
$$ds_0^2 = \frac{\mathcal{R}^2}{\ell^2}dt^2 - \frac{\ell^2}{\mathcal{R}^2}d\mathcal{R}^2 - \mathcal{R}^2 dv^2$$

PRD 101 (2020) 105002;
PoS(CORFU2021)240

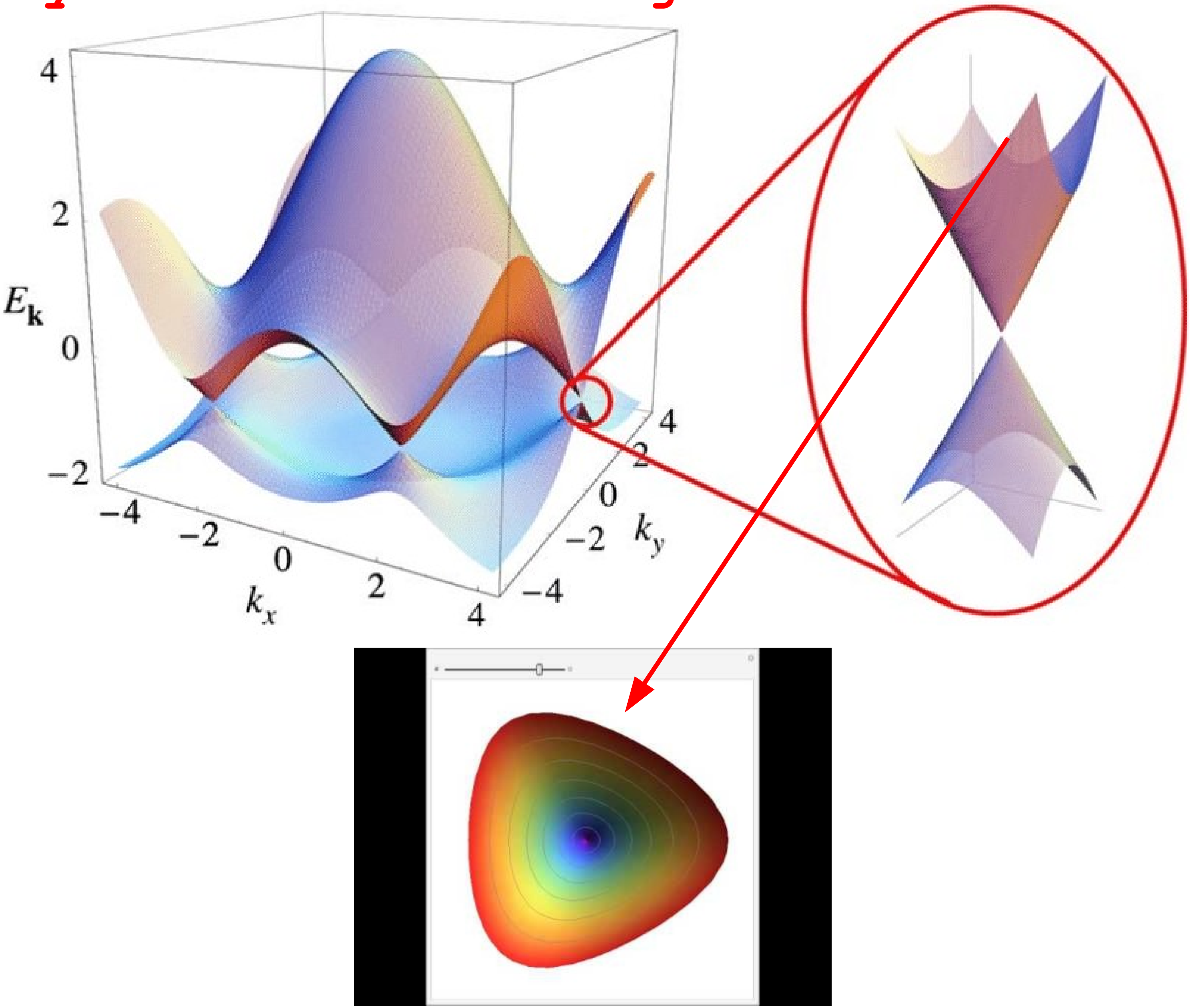
- Towards experiments

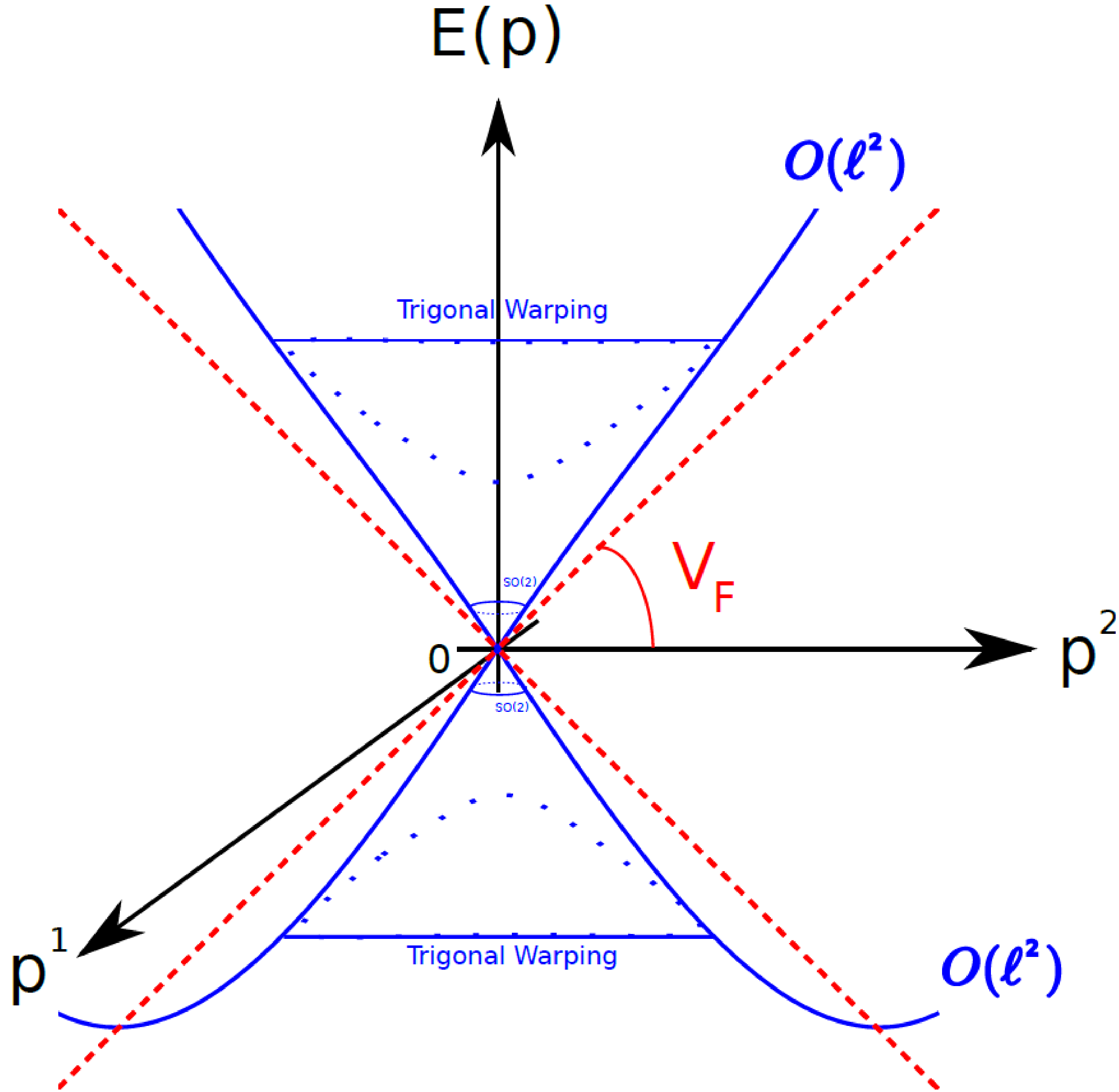


EPJ plus 127 (2012) 156; J Phys: Cond Matt 28 (2016) 13LT01 and ongoing

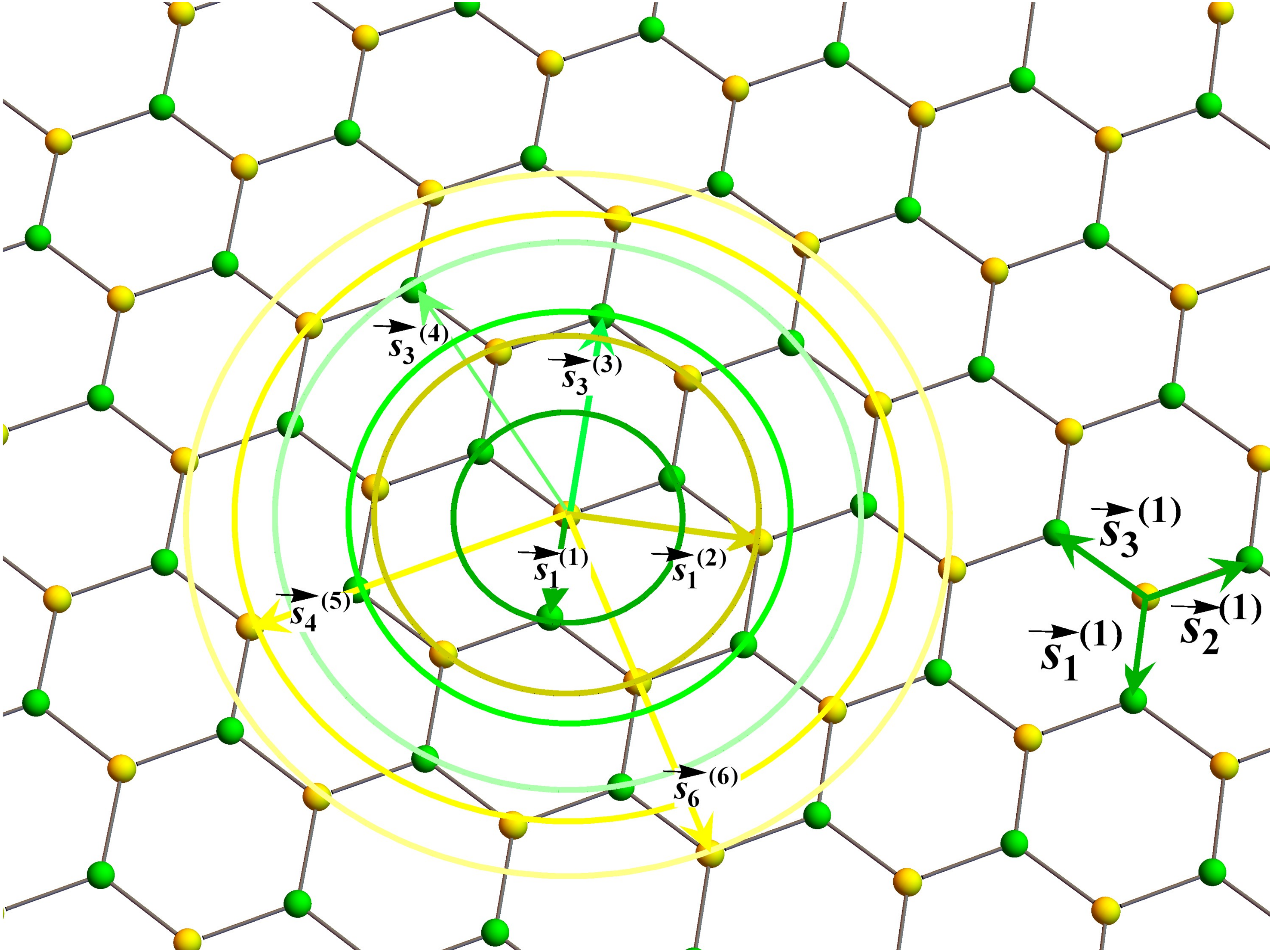


- **Beyond the linear regime**





E(p)
O(ℓ²)
Trigonal Warping
V_F
SO(2)
0
p²
SO(2)
p¹
Trigonal Warping
O(ℓ²)



$\vec{s}_3^{(4)}$
$\vec{s}_3^{(3)}$
$\vec{s}_1^{(1)}$
$\vec{s}_1^{(2)}$
$\vec{s}_4^{(5)}$
$\vec{s}_6^{(6)}$
$\vec{s}_3^{(1)}$
$\vec{s}_1^{(1)}$
$\vec{s}_2^{(1)}$

The full Hamiltonian

$$\mathcal{H}_{\vec{k}} = \sum_{m \in \mathbf{diag}} \eta_m \mathcal{F}_m(\vec{k})(a^*_{\vec{k}} a_{\vec{k}} + b^*_{\vec{k}} b_{\vec{k}}) + \left(\sum_{m \in \mathbf{off}} \eta_m \mathcal{F}^*_m(\vec{k}) a^*_{\vec{k}} b_{\vec{k}} + h.c. \right)$$

$$\mathcal{F}_m(\vec{k}) \equiv \sum_{i=1}^{n_m} e^{i\vec{k}\cdot\vec{s}_i^{(m)}} \qquad m = 1, 2, ..., \infty$$

The secular equation

$$\det\left(\mathcal{H}_{\vec{k}} - E\mathcal{S}_{\vec{k}}\right) = 0$$

$\mathcal{S}_{\vec{k}} = \sum_m \varsigma_m \mathcal{F}_m(\vec{k})$ **overlapping matrix elements**

$$\mathcal{F}_1(\vec{k}) = \sum_{i=1}^{3} e^{i\vec{k}\cdot\vec{s}_i^{(1)}} = e^{-i\ell k_y}\left[1 + 2e^{i\frac{3}{2}\ell k_y}\cos\left(\frac{\sqrt{3}}{2}\ell k_x\right)\right]$$

• The three 'layers' of graphene monolayer

The magic of $m = 2$: $\mathcal{F}_2 = |\mathcal{F}_1|^2 - 3$

$$E_{\pm} \simeq \eta_1 \left(\pm 0.97 |\mathcal{F}_1| - 0.15 |\mathcal{F}_1|^2 \pm 0.017 |\mathcal{F}_1|^3 \right)$$

$$\equiv V_F \left(\pm P_0 - A P_0^2 \pm B^2 P_0^3 \right)$$

where $V_F \equiv 0.97\, \eta_1 \ell/\hbar$, $A \equiv 0.15\, \ell/\hbar$, $B \equiv 0.13\, \ell/\hbar$ and we defined the *super-momenta*

$$\vec{P}_0 \equiv \frac{\hbar}{\ell} \left(\mathrm{Re}\mathcal{F}_1, \mathrm{Im}\mathcal{F}_1 \right)$$

So that the physics at $O(\ell^2)$ is given by

$$E_{\pm} = V_F \left(\pm |\vec{P}_0| - A\, |\vec{P}_0|^2 \right) \quad \left(* \right)$$

Expanding $\vec{P}_0$ one finds ($\tan\theta = p_y/p_x$)

$$|\vec{P}_0| \simeq \frac{3}{2} |\vec{p}| - \frac{3}{8} \ell |\vec{p}|^2 \cos(3\theta) - \frac{3}{64} \ell^2 |\vec{p}|^3 \cos^2(3\theta)$$

The dispersion relations $(*)$ also descend from the (effective) Hamiltonian

$$H_{super} = V_F \sum_{\vec{k}} \psi^{\dagger}_{\vec{k}} \left(\not{P}_0 - A \quad \not{P}_0 \not{P}_0 \right) \psi_{\vec{k}}$$

where $\not{P}_0 \equiv \vec{\sigma} \cdot \vec{P}_0$
We can go one more layer up, define *hyper-momenta*

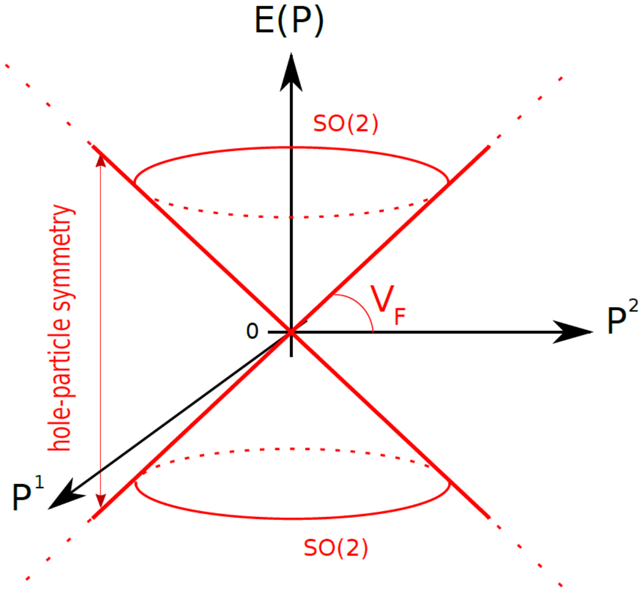
$$\vec{P} \equiv \vec{P}_0(1 - A|\vec{P}_0|)$$

and use $|\vec{P}_0| \to \vec{\sigma} \cdot \vec{P}_0$ **to obtain the (effective) Hamiltonian of the "even higher energy" layer**

$$H_{hyper} = V_F \sum_{\vec{k}} \psi^{\dagger}_{\vec{k}} \not{P} \psi_{\vec{k}}$$

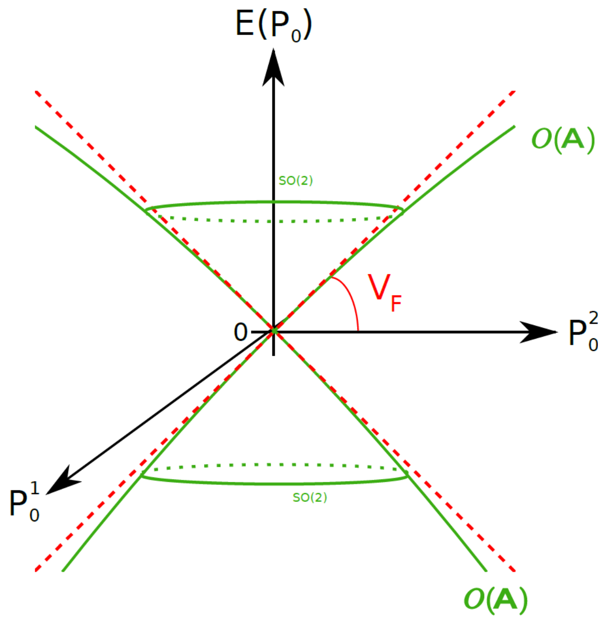
and

$$E_{\pm} = \pm V_F |\vec{P}|$$



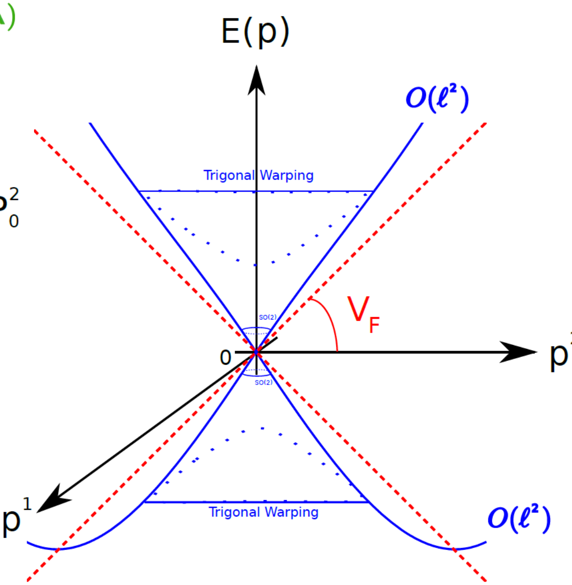
$$E_\pm = \pm V_F |\vec{P}|$$

$$H_{hyper} = V_F \sum_{\vec{k}} \psi^\dagger_{\vec{k}} \, \not{P} \psi_{\vec{k}}$$



$$E_\pm = V_F \left(\pm |\vec{P}_0| - A \, |\vec{P}_0|^2 \right)$$

$$H_{super} = V_F \sum_{\vec{k}} \psi^\dagger_{\vec{k}} \left(\not{P}_0 - A \, \not{P}_0 \, \not{P}_0 \right) \psi_{\vec{k}}$$



$$E_\pm = v_F \left(\pm |\vec{p}| \mp \frac{\ell}{4} |\vec{p}|^2 \cos 3\theta \mp \frac{\ell^2}{64} |\vec{p}|^3 (7 + \cos 6\theta) - \frac{3}{2} A \, |\vec{p}|^2 + \frac{3}{4} A \, \ell \, |\vec{p}|^3 \cos 3\theta \right)$$

$$
\begin{aligned}
H \;=\; & v_F \sum_{\vec{p}} \psi^\dagger_{\vec{p}} \Bigg[\sigma_1 \left(p_1 - \frac{\ell}{4}(p_1^2 - p_2^2) - \frac{\ell^2}{8} p_1 (p_1^2 + p_2^2) \right) \\
+ \; & \sigma_2 \left(p_2 + \frac{\ell}{2} p_1 p_2 - \frac{\ell^2}{8} p_2 (p_1^2 + p_2^2) \right) \\
- \; & \frac{3}{2} A \left((p_1^2 + p_2^2) - \frac{\ell}{2} p_1^3 + \frac{3\ell}{2} p_1 p_2^2 \right) \Bigg] \psi_{\vec{p}} \,.
\end{aligned}
$$

$P_0(p)$

$P(p)$

$$
E_\pm = v_F \left(\underbrace{\underbrace{\pm|\vec{p}| \mp \frac{\ell}{4}|\vec{p}|^2 \cos 3\theta \mp \frac{\ell^2}{64}|\vec{p}|^3 (7 + \cos 6\theta)}_{P_0(p)} - \frac{3}{2} A\, |\vec{p}|^2 + \frac{3}{4} A\, \ell\, |\vec{p}|^3 \cos 3\theta}_{P(p)} \right)
$$

The standard variables (x,p) are under control $(\hbar = 1)$

$$[x_i, p_j] = \imath\delta_{ij}\,,\ \ [x_i, x_j] = 0 = [p_i, p_j]$$

The supermomenta $P_0^i(p)$ are given. The hypermomenta $P^i(P_0(p))$ are given.

What are the supercoordinates X_0^i and the hypercoordinates X^i?

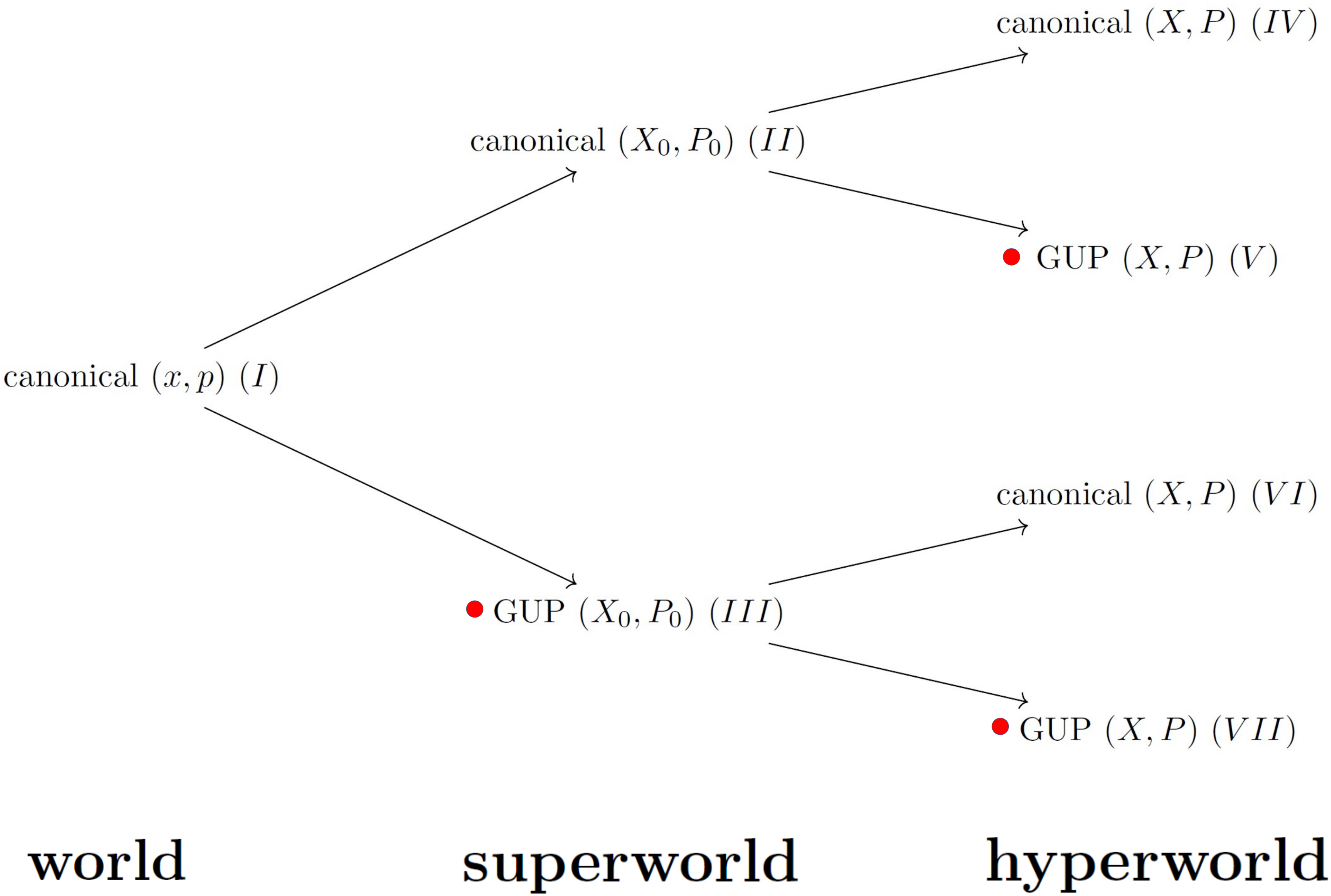
We need

$$X_0^i(x,p)$$

and

$$X^i(x,p)$$

- **Three GUPs ($[X^i, X^j] = 0$)**



The most remarkable ladder is $(I) \to (III) \to (VII)$

$$\underline{x^i = X_0^i = X^i}$$

First step $(I) \to (III)$

$$[X_0^i, P_0^j] = \imath F^{ij}(\vec{P}_0)$$

where

$$F^{ij}(\vec{P}_0) = \delta^{ij} + \frac{1}{2}\ell \begin{pmatrix} -P_0^1 & P_0^2 \\ P_0^2 & P_0^1 \end{pmatrix} - \frac{1}{2}\ell^2 \begin{pmatrix} (P_0^1)^2 & P_0^1 P_0^2 \\ P_0^1 P_0^2 & (P_0^2)^2 \end{pmatrix}$$

Second step $(III) \to (VII)$

$$[X^i, P^j] \;=\; \imath\, \mathcal{F}^{i\,j}(\vec{P})$$

where

$$\mathcal{F}^{i\,j}(\vec{P}) \;\equiv\; \imath F^{i\,k}(\vec{P}) \left[\delta^{kj} - A\,|\vec{P}| \left(\delta^{kj} + \frac{P^k\,P^j}{|\vec{P}|^2}\right) - A^2\,|\vec{P}|^2 \left(\delta^{kj} + \frac{P^k\,P^j}{|\vec{P}|^2}\right)\right]$$

Also noticeable is the "canonical ladder" $(I) \to (II) \to (IV)$

First step $(I) \to (II)$ **for** $X_0(x,p)$ **given by**

$$\begin{aligned} X_0^1 &= \left[1 + \frac{\ell}{2}p^1 + \frac{\ell^2}{8}\left(5(p^1)^2 + 3(p^2)^2\right)\right] x^1 + \left[-\frac{\ell}{2}p^2 + \frac{\ell^2}{4}p^1 p^2\right] x^2 \\ X_0^2 &= \left[-\frac{\ell}{2}p^2 + \frac{\ell^2}{4}p^1 p^2\right] x^1 + \left[1 - \frac{\ell}{2}p^1 + \frac{\ell^2}{8}\left(3(p^1)^2 + 5(p^2)^2\right)\right] x^2 \end{aligned} \qquad (\dagger)$$

Second step $(II) \to (IV)$ **for** $X(X_0, P_0)$ **given by**

$$X^i = \frac{X_0^i}{1 - A\,|\vec{P}_0|} + \frac{A(X_0^j P_{0j})P_0^i}{|\vec{P}_0|\,(1 - A\,|\vec{P}_0|)(1 - 2A\,|\vec{P}_0|)}$$

The $X^i(x,p)$ are then

$$X^1 = x^1 + \frac{1}{2}\,\ell\,(p^1x^1 - p^2x^2) + \frac{1}{8}\,\ell^2\left(5(p^1)^2x^1 + 2p^1p^2x^2 + 3(p^2)^2x^1\right)$$
$$+ \frac{A\left(2(p^1)^2x^1 + p^1p^2x^2 + (p^2)^2x^1\right)}{|\vec{p}|} + A^2\left(4(p^1)^2x^1 + 3p^1p^2x^2 + (p^2)^2x^1\right)$$
$$+ \frac{A\,\ell\,\left(2(p^1)^5x^1 - 4(p^1)^4p^2x^2 + 3(p^1)^3(p^2)^2x^1 - 9(p^1)^2(p^2)^3x^2 + 5p^1(p^2)^4x^1 - (p^2)^5x^2\right)}{4|\vec{p}|^3}$$

$$X^2 = x^2 - \frac{1}{2}\,\ell\,(p^1\,x^2 + p^2\,x^1) + \frac{1}{8}\,\ell^2\left(3(p^1)^2x^2 + 2p^1\,p^2x^1 + 5(p^2)^2x^2\right)$$
$$+ \frac{A\left((p^1)^2x^2 + p^1\,p^2x^1 + 2(p^2)^2x^2\right)}{|\vec{p}|} + A^2\left((p^1)^2x^2 + 3p^1\,p^2x^1 + 4(p^2)^2x^2\right)$$
$$- \frac{A\,\ell\,\left(3(p^1)^5x^2 - 2(p^1)^4p^2x^1 + (p^1)^3(p^2)^2x^2 + 5(p^1)^2(p^2)^3x^1 + 2p^1\,(p^2)^4x^2 + 3(p^2)^5x^1\right)}{4|\vec{p}|^3}$$

Then we have the two "middle-way" ladders

$$(I)\ \to\ (II)\ \to\ (V)$$

$$[X^i, P^j] = \mathrm{i}\left[\delta^{ij} - A\,|\vec{P}|\,\left(\delta^{ij} + \frac{P^i P^j}{|\vec{P}|^2}\right) - A^2\,|\vec{P}|^2\,\left(\delta^{ij} + \frac{P^i P^j}{|\vec{P}|^2}\right)\right]$$

$$X^i = X^i_0(x, p) \ \textbf{given in}\ (\dagger)$$

$$(I)\ \to\ (III)\ \to\ (VI)$$

$$[X^i, P^j] = \mathrm{i}\,\delta^{ij}$$

$$X^1 = G^{11}(\vec{p})\,x^1 + G^{12}(p)\,x^2$$

$$X^2 = G^{21}(\vec{p})\,x^1 + G^{22}(p)\,x^2$$

- **The fourth case, $[X^i, X^j] \neq 0$**

We can consider another, more general, case

$$[X^i, P^j] = i\hbar\,\mathcal{F}^{i\,j}(\vec{P})\ , \quad [X^i, X^j] = i\mathcal{G}^{i\,j}(\vec{P})\ , \quad [P^i, P^j] = 0$$

Besides zero, the simplest choice is

$$\mathcal{G}^{ij} = L^2\,\epsilon^{ij}$$

Since all calculations are $O(\ell^2)$

$$L(\ell) = a\,\ell$$

Therefore we have

$$[X^i, X^j] = i\,\theta^{ij}$$

where

$$\theta^{ij} = \ell^2\,\epsilon^{ij}$$

and

$$X_1 = x_1 - \frac{\ell^2}{2\hbar}\,p_2\,, \qquad X_2 = x_2 + \frac{\ell^2}{2\hbar}\,p_1$$

i.e.

$$X^i = x^i - \frac{1}{2\hbar}\,\theta^{ij} p_j$$

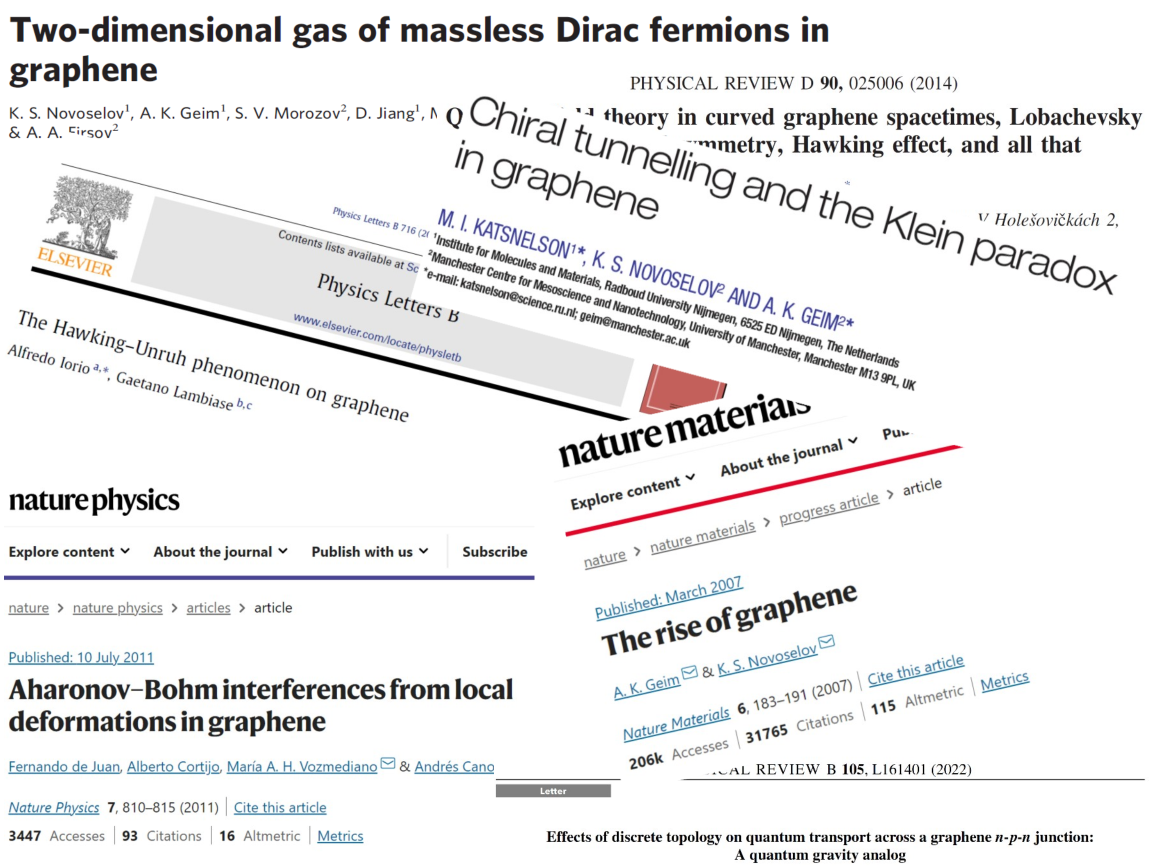
(Bopp's shift).

• What to do with this?

All the 'fundamental' physics realized on graphene using

$$H = v_F \sum_{\vec{p}} \psi^\dagger_{\vec{p}} \vec{\sigma} \cdot \vec{p} \psi_{\vec{p}}$$

(and $m \neq 0$, $\vec{p} \to \vec{p} + \vec{A}$, $\vec{p} \to \vec{p} + \vec{\Omega}(\omega, \kappa)$, etc...)



can be GUP-corrected

15 April 1999
PHYSICS LETTERS B
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Physics Letters B 452 (1999) 39–44

Generalized uncertainty principle in quantum gravity from micro-black hole gedanken experiment

Fabio Scardigli [1]

PHYSICAL REVIEW D **101**, 105002 (2020)

Generalized uncertainty principle in three-dimensional gravity and the BTZ black hole

Alfredo Iorio, [1,*] Gaetano Lambiase, [2,†] Pablo Pais, [1,3,‡] and Fabio Scardigli [4,5,§]

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Regular Article - Theoretical Physics

Phenomenology of GUP stars

Luca Buoninfante[1,a], Gaetano Lambiase[2,3,b], Giuseppe Gaetano Luciano[2,3,c], Luciano Petruzziello[2,4,d]

Eur. Phys. J. C (2021) 81:982
https://doi.org/10.1140/epjc/s10052-021-09795-1
Regular Article - Theoretical Physics

Generalized uncertainty principle: from the harmonic oscillator a QFT toy model

Pasquale Bosso[1,a], Giuseppe Gaetano Luciano[3,2,b]

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ARTICLE
https://doi.org/10.1038/s41467-021-24711-7 OPEN

Quantum gravitational decoherence from fluctuating minimal length and deformation parameter at the Planck scale

Luciano Petruzziello [1,2] & Fabrizio Illuminati [1,2]

by simply including higher order terms

$$\begin{aligned} H \; = \; & v_F \sum_{\vec{p}} \psi^\dagger_{\vec{p}} \Bigg[\sigma_1 \left(p_1 - \frac{\ell}{4}(p_1^2 - p_2^2) - \frac{\ell^2}{8} p_1 (p_1^2 + p_2^2) \right) \\ & + \; \sigma_2 \left(p_2 + \frac{\ell}{2} p_1 p_2 - \frac{\ell^2}{8} p_2 (p_1^2 + p_2^2) \right) \\ & - \; \frac{3}{2} A \left((p_1^2 + p_2^2) - \frac{\ell}{2} p_1^3 + \frac{3\ell}{2} p_1 p_2^2 \right) \Bigg] \psi_{\vec{p}} \, . \end{aligned}$$

$$[X_0^i, P_0^j] = \imath F^{ij}(\vec{P}_0)$$

$$[X^i, P^j] \; = \; \imath \, \mathcal{F}^{ij}(\vec{P})$$

and by using the operational recipe:

$$\begin{tikzcd}[column sep=huge, row sep=huge]
(\vec{x},\vec{p});H(x,p) \arrow[r, "\text{experiment}"] \arrow[d, "\text{analog}"'] & E_n(H(x,p)) \arrow[d, leftrightarrow, "\text{comparison}"] \\
(\vec{X},\vec{P});H(X,P) \arrow[r, "\text{calculation}"] & E_n(H(X,P))
\end{tikzcd}$$

where, e.g., E_n **is the energy spectrum**