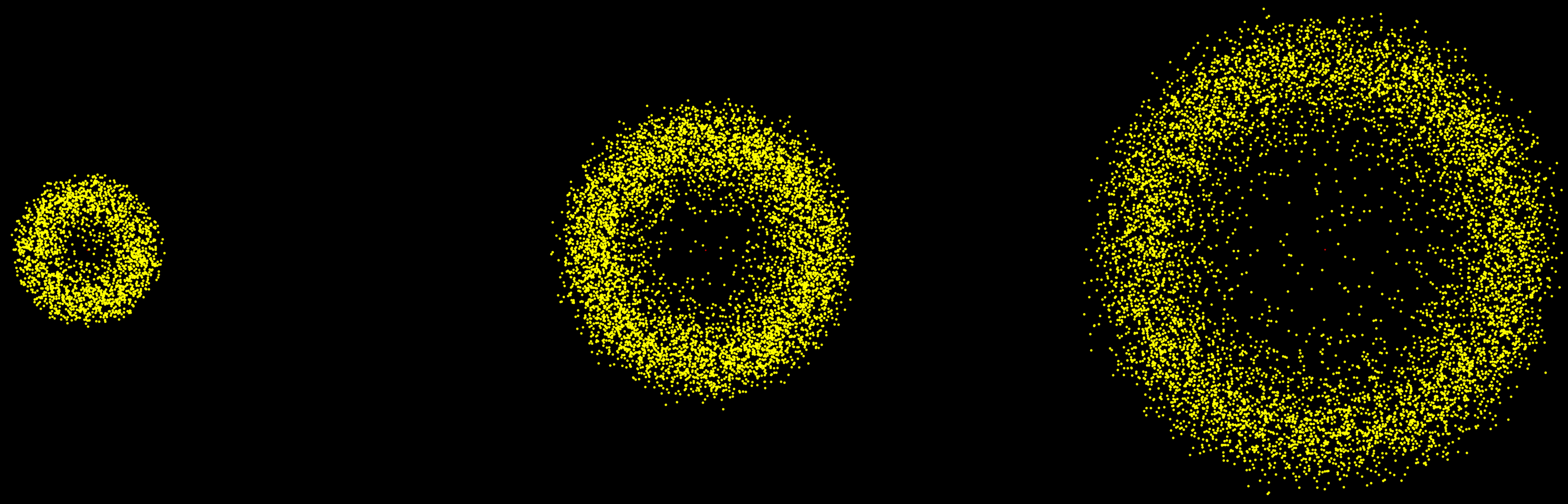
Robustness, sensitivity and pseudospectra around higher order exceptional points



K. G. Makris
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Analytic and Algebraic methods in Physics XIX, 8 September 2022, Prague, Czech Republic

Synopsis

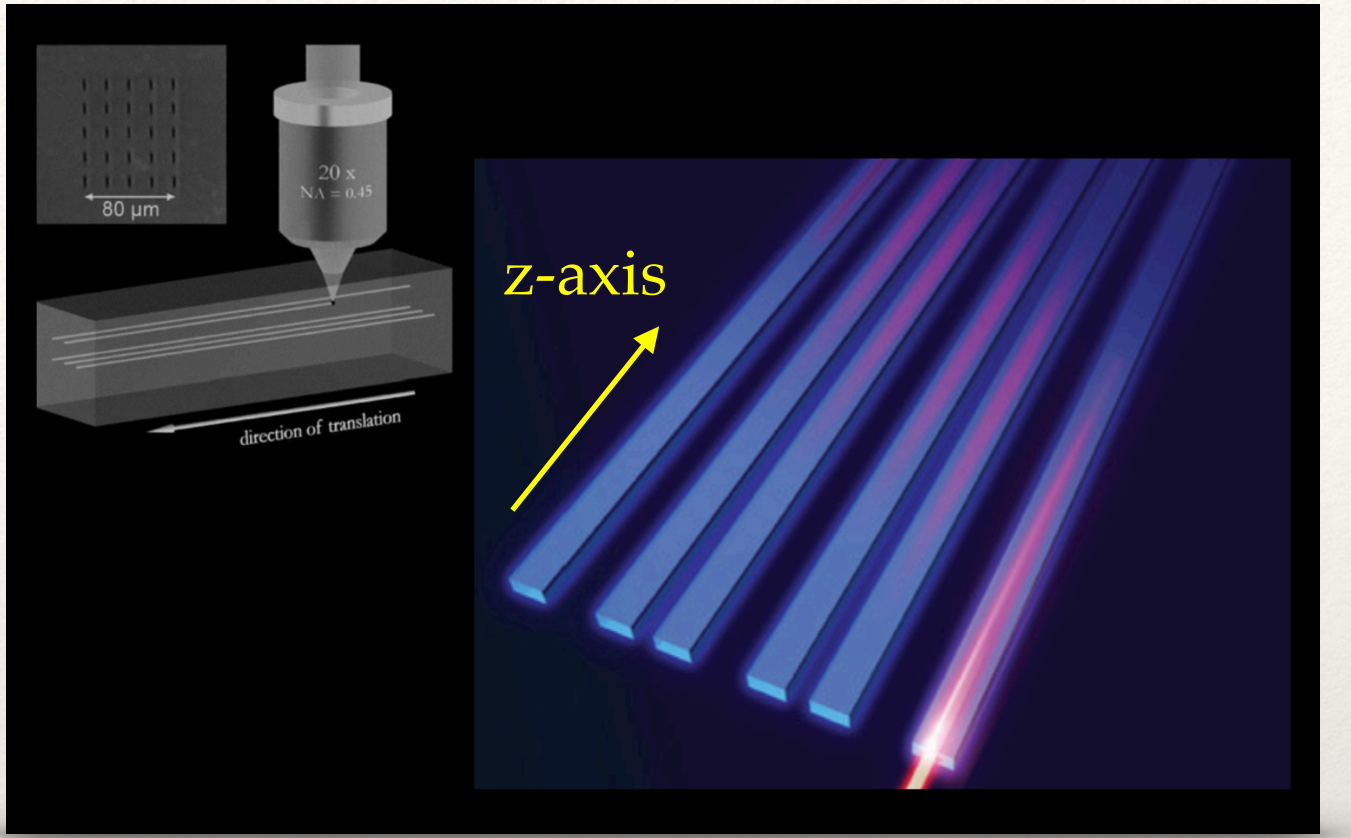
- Physical Motivation and non-Hermitian Photonics
- Non-Hermitian Degeneracies: Exceptional points
- **Part A: Higher-Order Exceptional points in lattices**
- **Part B: Extreme behaviour around Exceptional Points**
- Conclusions

Physical Motivation

Non-Hermitian Photonics

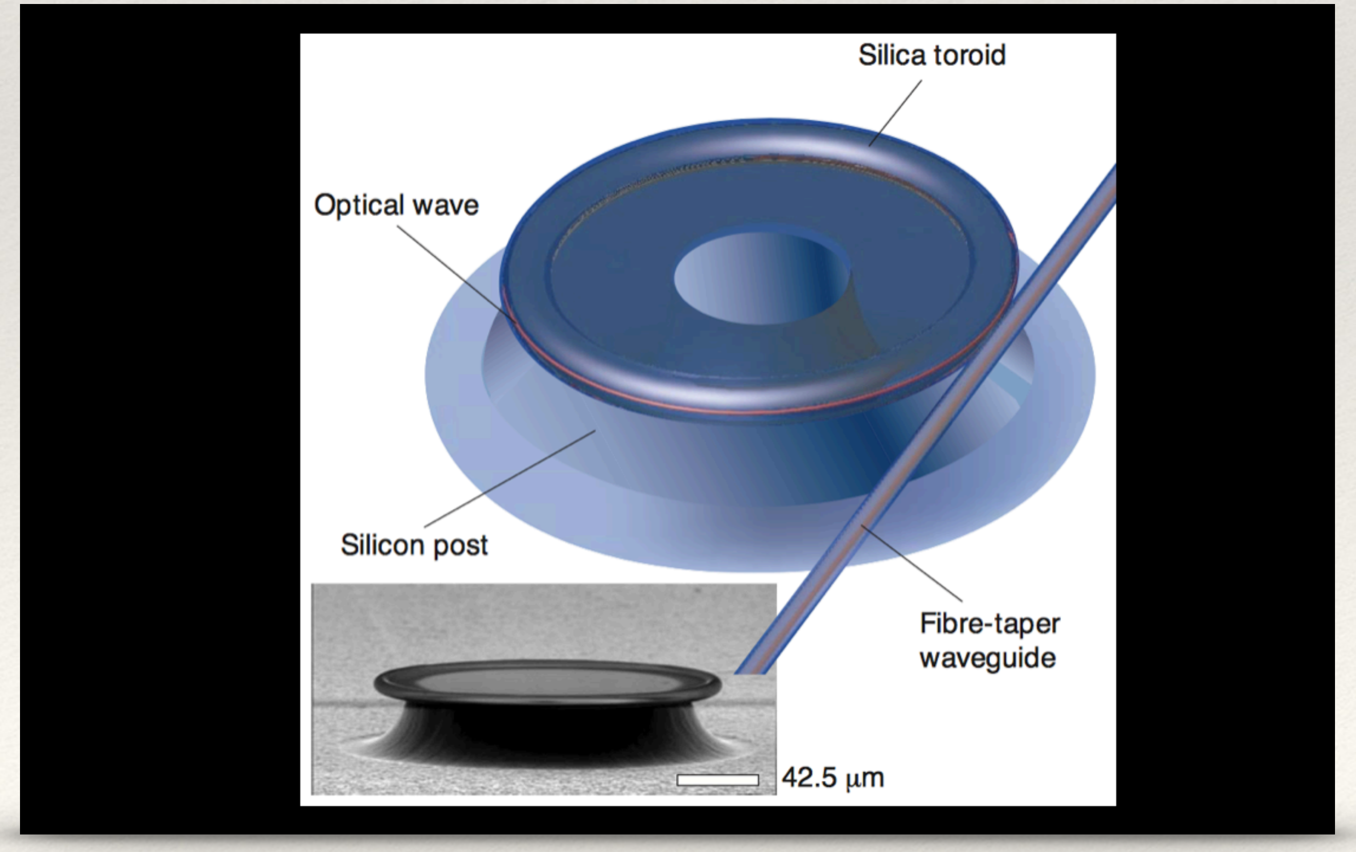
Photons on chip

Guiding light in integrated systems



Waveguides

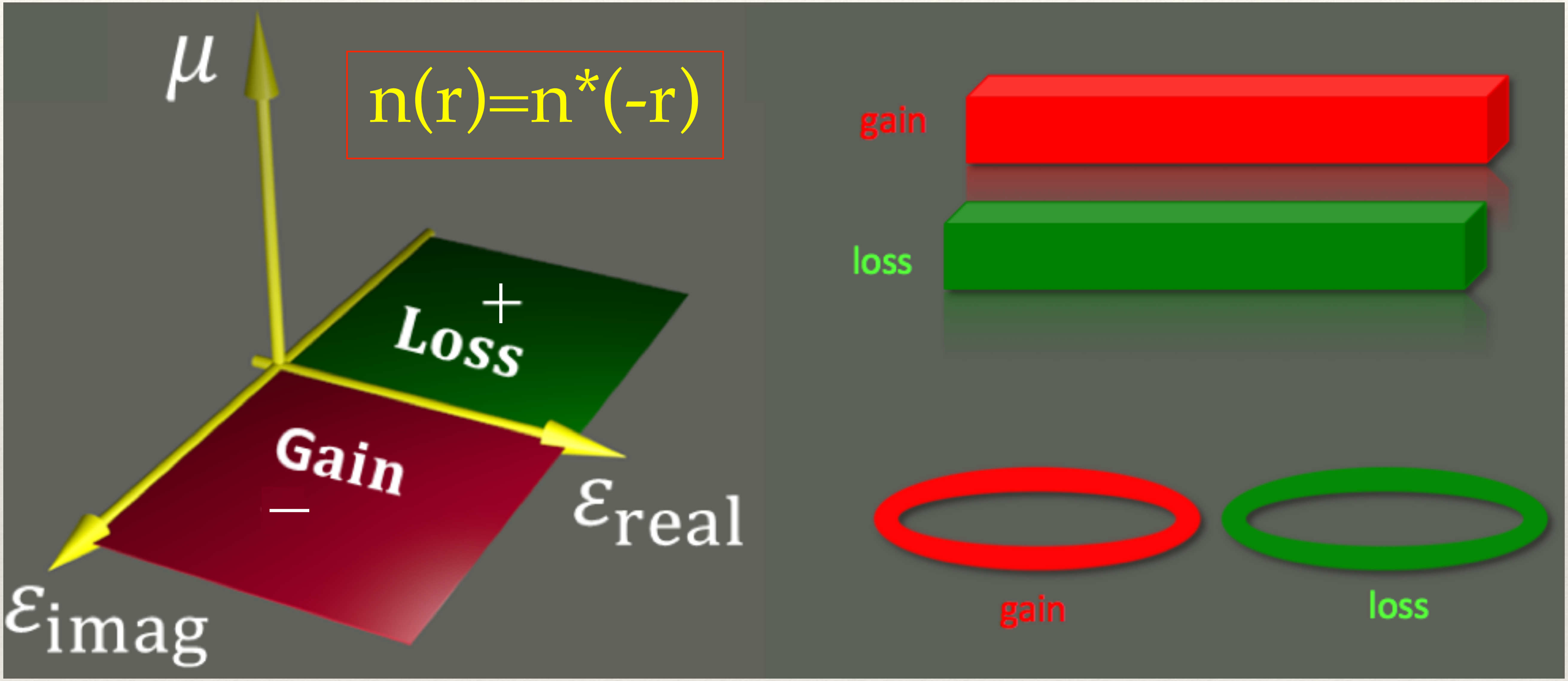
Real refractive index



Cavities

Complex Refractive Index

C. M. Bender, and S. Boettcher, Phys. Rev. Lett. 80 5243 (1998)
K. G. Makris, R. El-Ganainy, D. N. Christodoulides, and Z. H. Musslimani, Phys. Rev. Lett. 100, 103904 (2008)
C. Ruter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev and D. Kip, Nat. Phys. 6, 192 (2010)

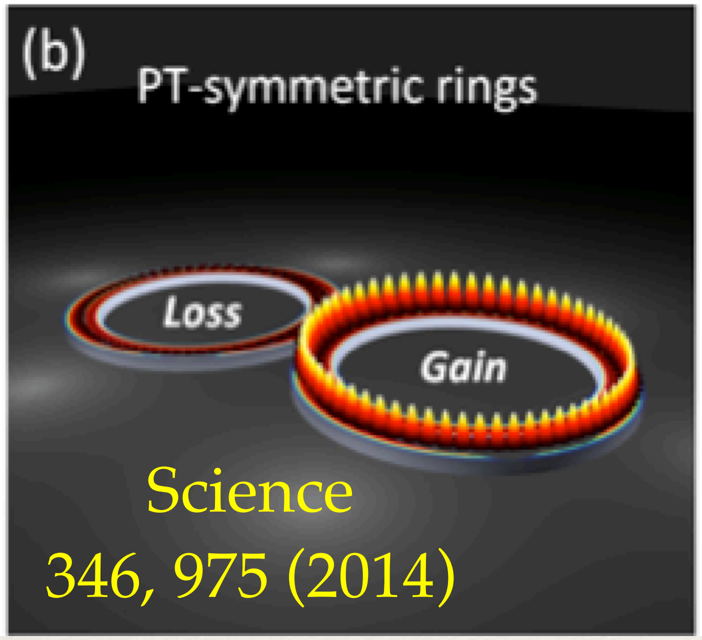


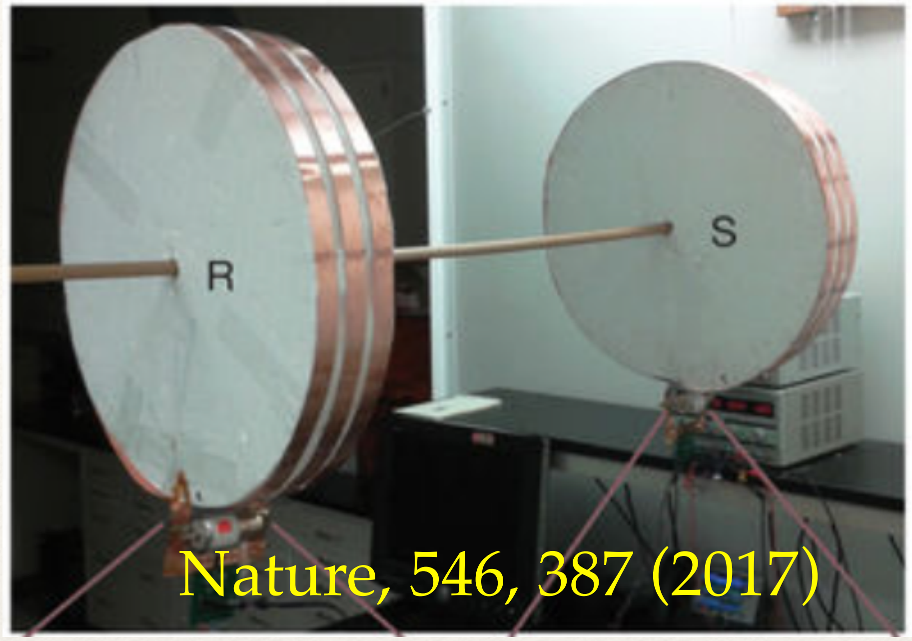
$\varepsilon = \varepsilon_R + i\varepsilon_I$ Permittivity

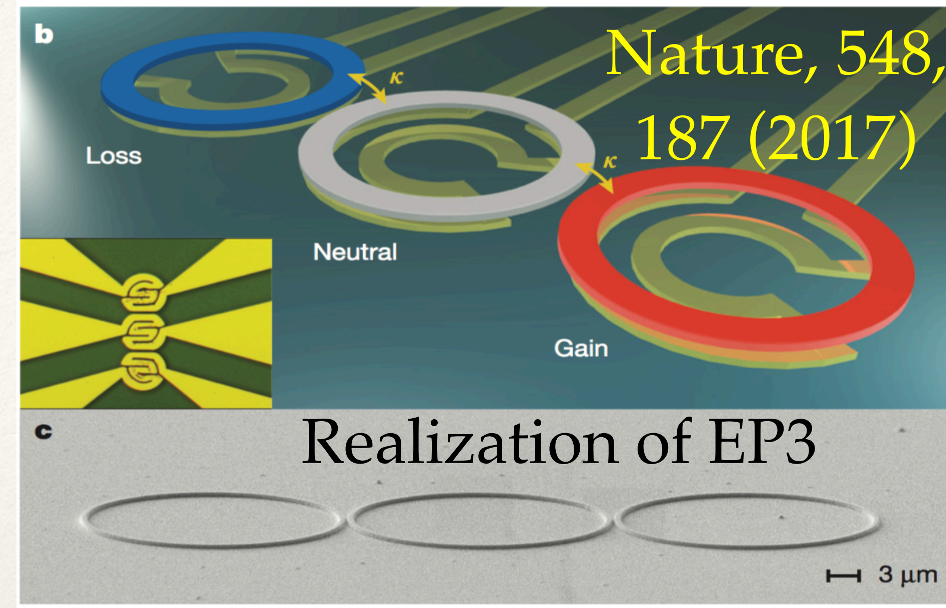
$n = n_R + in_I$ Refractive index

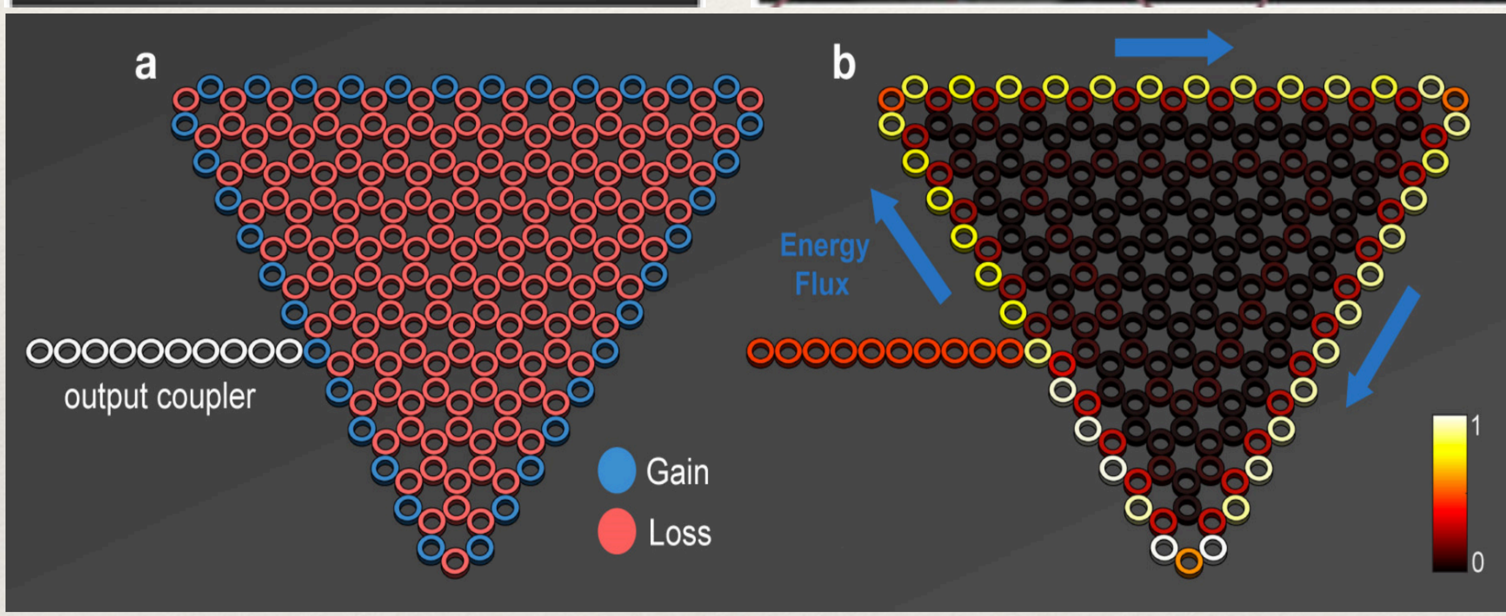
$$n_I, \varepsilon_I = \begin{cases} > 0, & loss \\ < 0, & gain \end{cases}$$

Recent developments









[Topological insulator laser: theory]()
Science 359 (6381), eaar4003, (2018).

[Topological insulator laser: Experiments]()
Science 359 (6381), eaar4005 (2018).

nature physics

REVIEW ARTICLES

PUBLISHED ONLINE: 5 JANUARY 2017 | DOI: 10.1038/NPHYS4323

Nat. Phys. 14, 11 (2018)

Non-Hermitian physics and PT symmetry

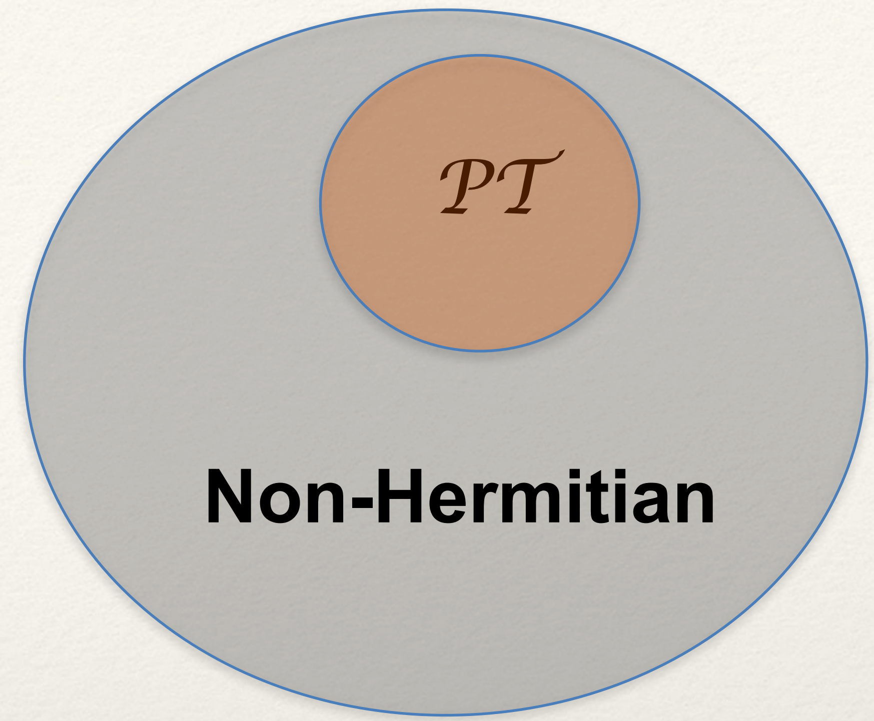
Ramy El-Ganainy[1], Konstantinos G. Makris[2], Mercedeh Khajavikhan[3], Ziad H. Musslimani[4], Stefan Rotter[5] and Demetrios N. Christodoulides[3]*

Exceptional Points

Non-Hermitian Degeneracies

2x2 Non-Hermitian Matrix

- **Complex spectrum**
- **Non-orthogonality**
- **Exceptional points**
- **Energy in not conserved**
- **Ultra-sensitivity**



$$M = \begin{pmatrix} \delta - ig_1 & \kappa \\ \kappa & ig_2 \end{pmatrix}$$

$$M \neq M^\dagger$$

Non-Hermitian matrix

$$M\left|\phi_n\right\rangle = \lambda_n\left|\phi_n\right\rangle \qquad \left\langle\tilde{\phi}_m\middle|\phi_n\right\rangle = \delta_{n,m}$$

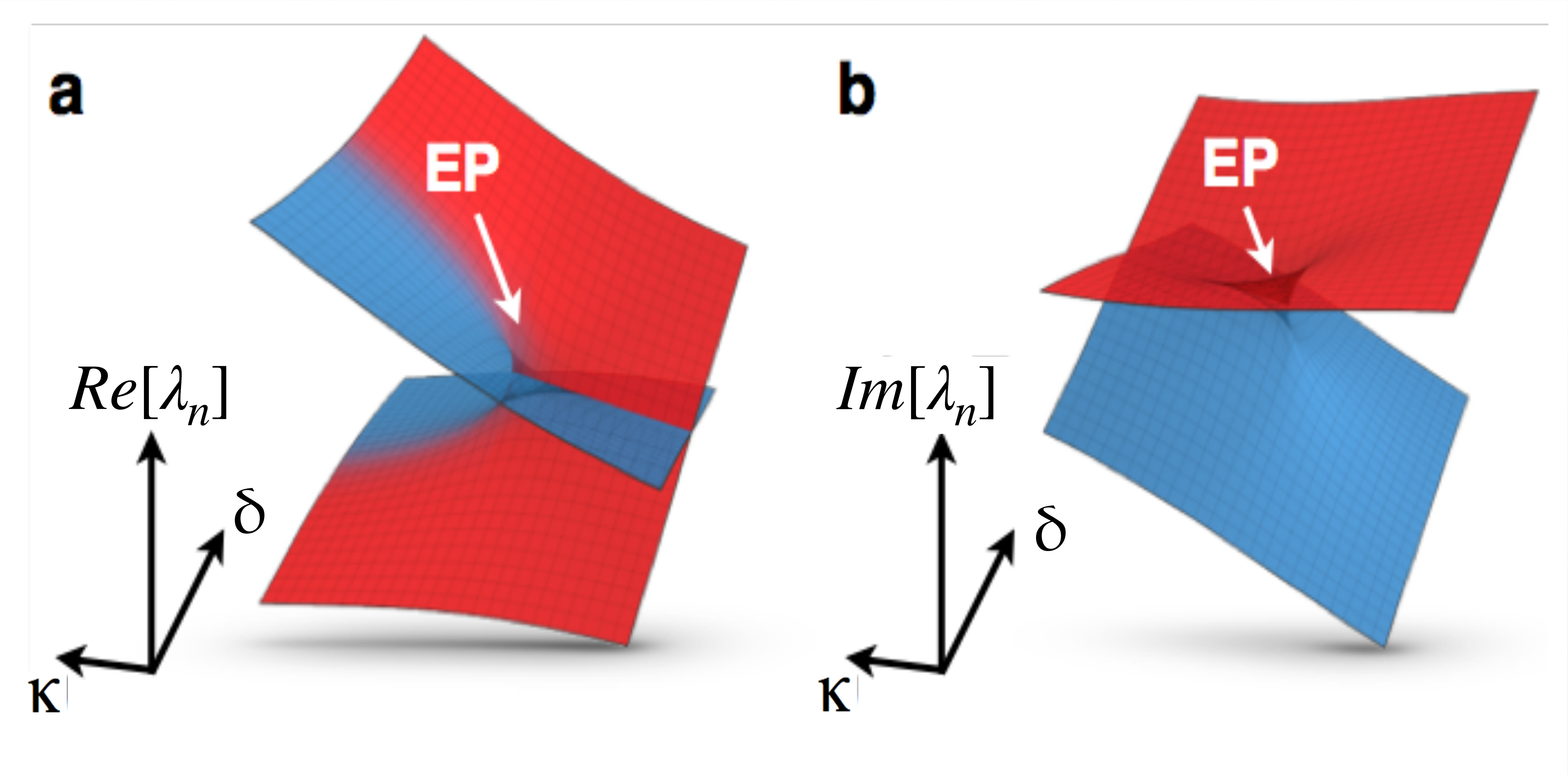
Bi-orthogonality Condition

$$\boxed{\lambda_{1,2} = \frac{1}{2}\left\{\delta - i\left(g_1 - g_2\right) \pm \sqrt{\delta^2 + 4\kappa^2 - \left(g_1 + g_2\right)^2 - 2i\delta\left(g_1 + g_2\right)}\right\}}$$

A non-Hermitian matrix has complex eigenvalues in general

I. Rotter, J. Phys. A: Math. Theor. 42,153001 (2009)

Exceptional Points



$$\delta = 0, \quad \kappa = \frac{|g_1 + g_2|}{2} \Rightarrow \quad \lambda_1 = \lambda_2 = -i\frac{g_1 - g_2}{2}$$

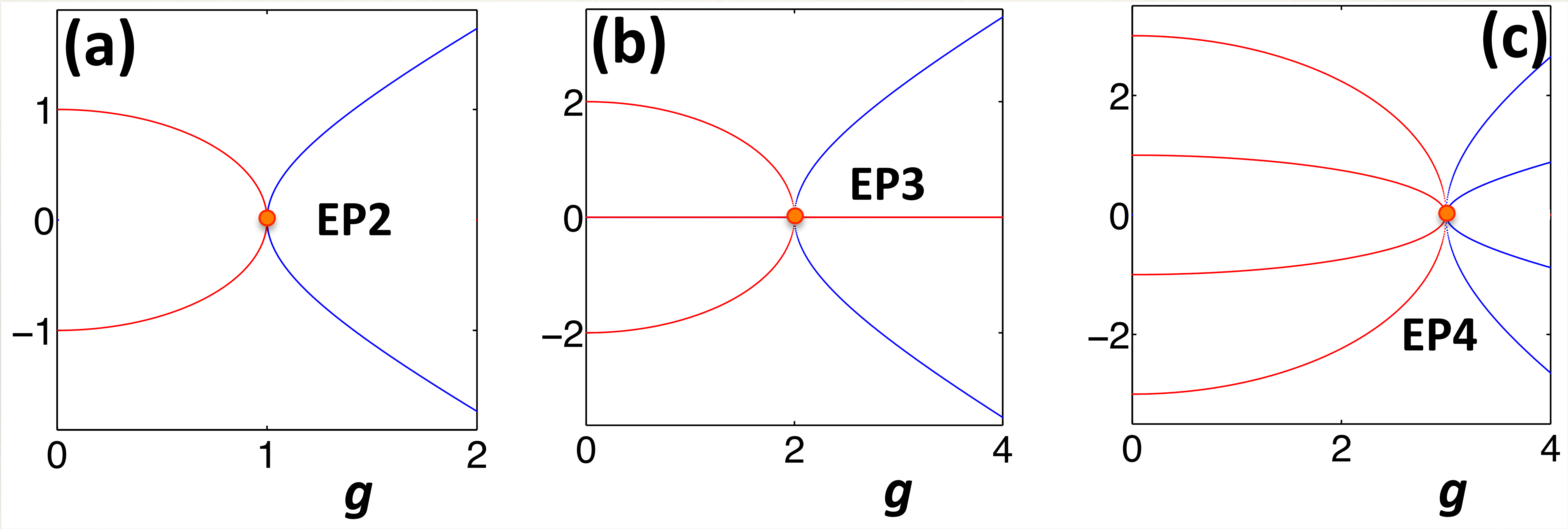
Eigenvalues and eigenvectors coalesce: Exceptional Point (EP)

Higher order EPs

$$M_2 = i\begin{pmatrix} ig & \kappa \\ \kappa & -ig \end{pmatrix}$$

$$M_3 = i\begin{pmatrix} -2ig & \sqrt{2}\kappa & 0 \\ \sqrt{2}\kappa & 0 & \sqrt{2}\kappa \\ 0 & \sqrt{2}\kappa & 2ig \end{pmatrix}$$

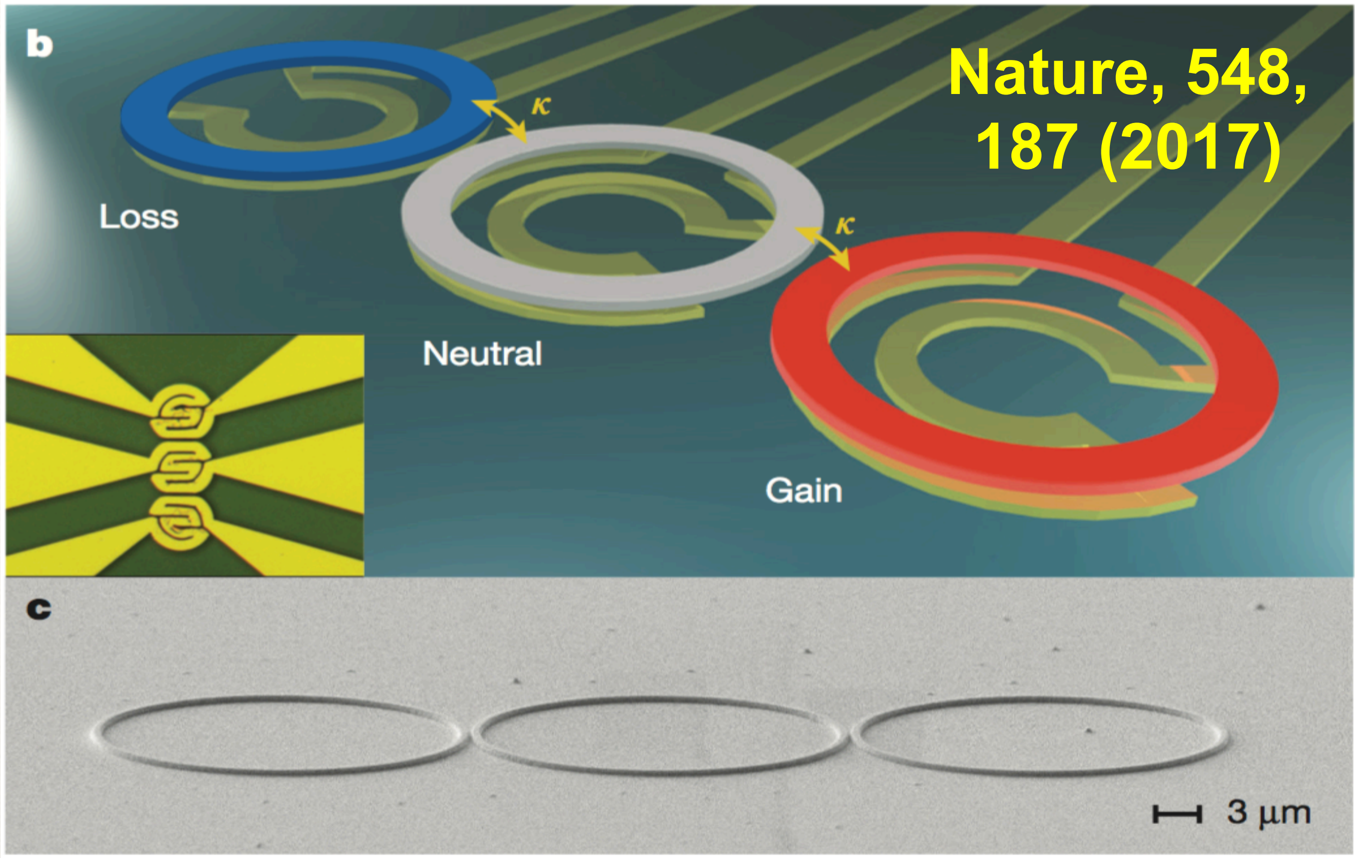
$$M_4 = i\begin{pmatrix} -2ig & \sqrt{3}\kappa & 0 & 0 \\ \sqrt{3}\kappa & -ig & \sqrt{3}\kappa & 0 \\ 0 & \sqrt{3}\kappa & ig & \sqrt{3}\kappa \\ 0 & 0 & \sqrt{3}\kappa & 2ig \end{pmatrix}$$



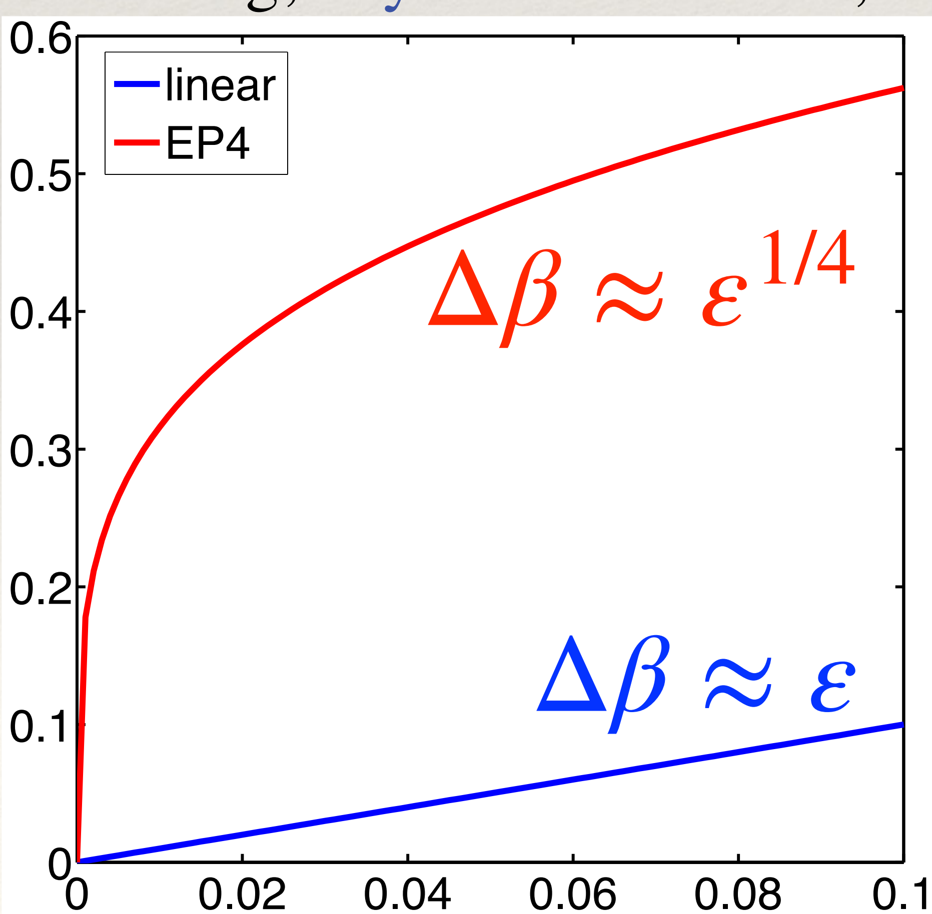
E. M. Graefe, U. Günther, H. J. Korsch, and A. E. Niederle, J. Phys. A **41**, 255206 (2008).

D. Gilles and E.M.Graefe, J. Phys. A **45**, 025303 (2012). W. D. Heiss and G. Wunner, J. Phys. A **49**, 495303 (2016).

Amplification and Sensitivity close to EPNs

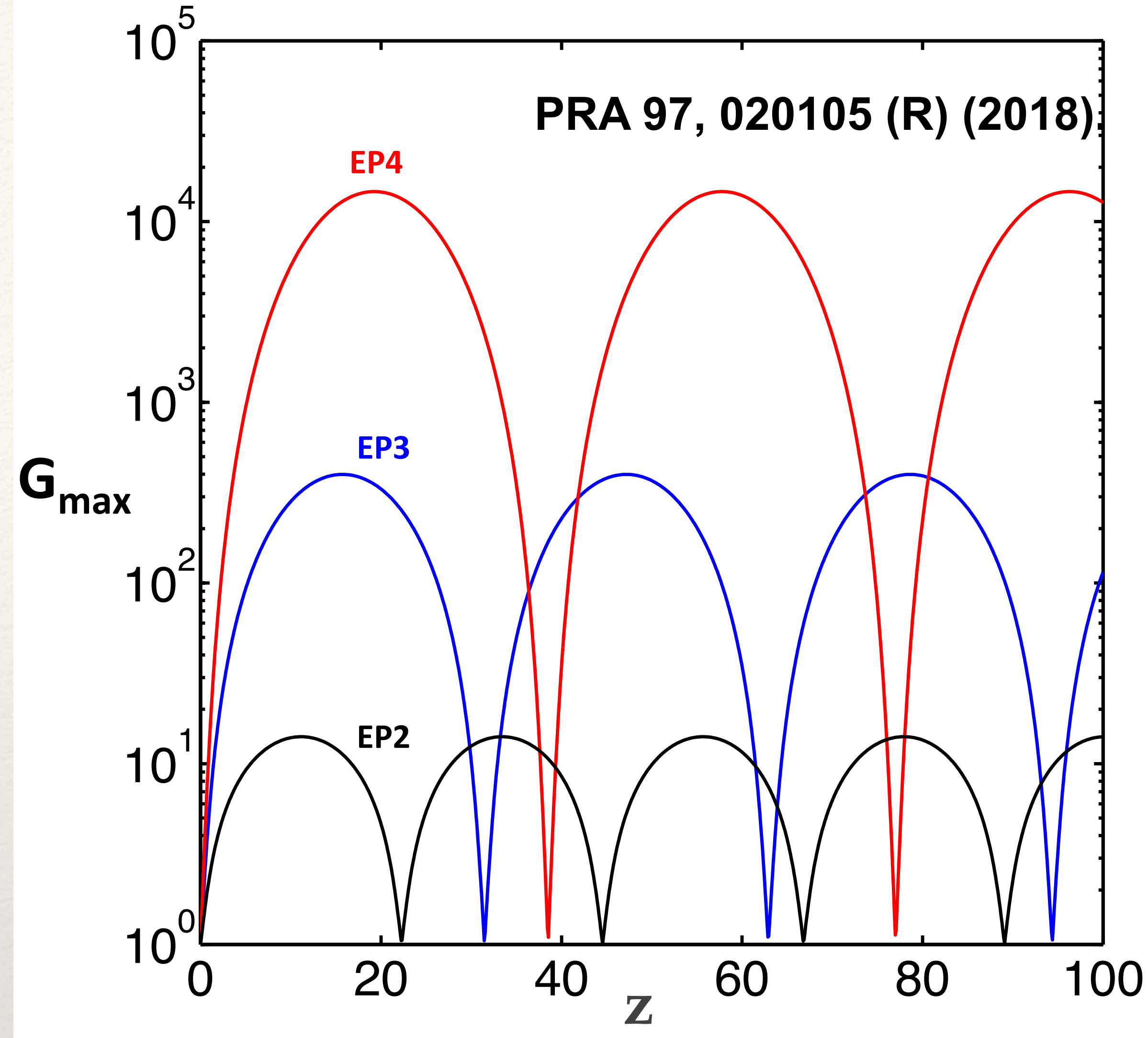


J. Wiersig, Phys. Rev. Lett. **112**, 203901 (2014)



$$\Delta\beta \approx \varepsilon^{1/N}$$

Sensitivity at EPNs



$$\tilde{g} \equiv g/\kappa$$

$$G_{max} = max_z \|e^{zH}\| = (\frac{1+\tilde{g}}{1-\tilde{g}})^{N-1}$$

Maximal amplification at EPNs

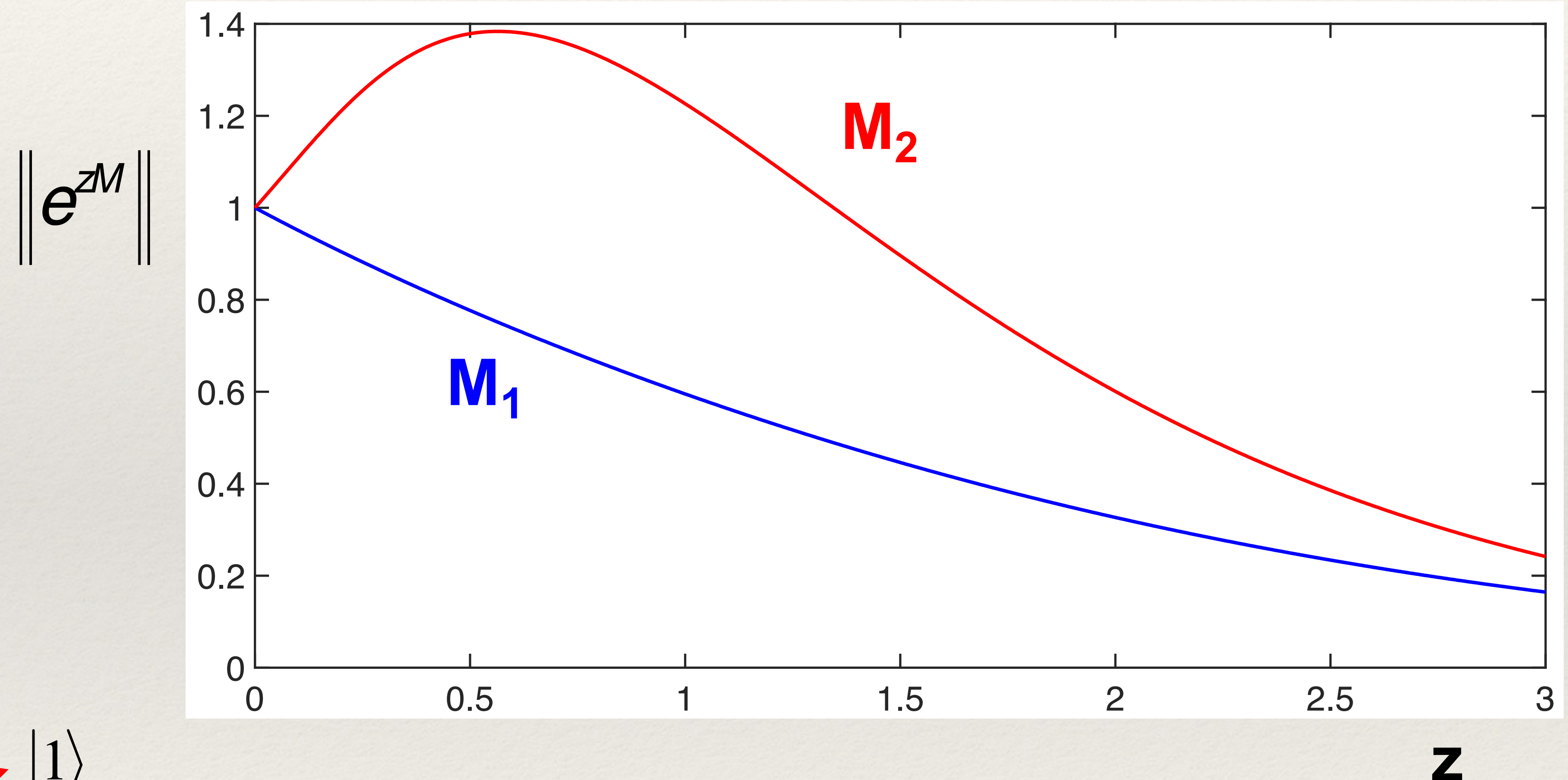
Can spectrum alone determine dynamics?

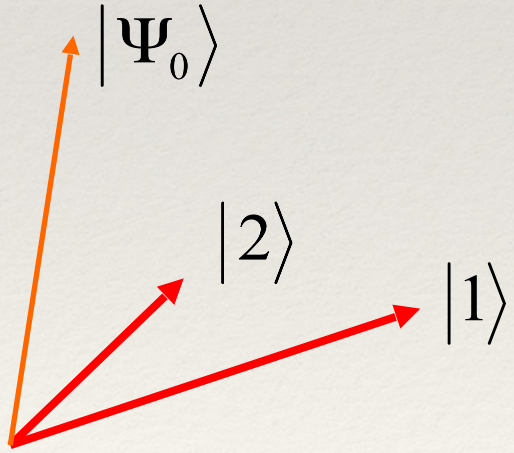
$$M_1 = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$$

$$\boxed{\lambda_{1,2} = -1, -1 < 0}$$

$$M_2 = \begin{pmatrix} -1 & 5 \\ 0 & -2 \end{pmatrix}$$

$$\boxed{\lambda_{1,2} = -1, -2 < 0}$$





$$|\Psi_0\rangle = |1\rangle \exp(i\beta_1 z) + |2\rangle \exp(i\beta_2 z)$$

$$\langle \Psi_0 | \Psi_0 \rangle = \langle 1|1\rangle + \langle 2|2\rangle + \langle 2|1\rangle \exp(i\Delta\beta\, z) + cc$$

Pseudospectrum

$\|x\|$ **Norm of a vector**

$$\|M\| \equiv \max_x \frac{\|Mx\|}{\|x\|}$$ **Norm of a matrix**

$$\sigma(H) \equiv$$ **Eigenvalue spectrum**

Pseudospectrum

$$\boxed{\sigma_\varepsilon(H) \equiv \bigcup_{i=1,||E_i||<\varepsilon}^{M} \sigma(H + E_i)}$$

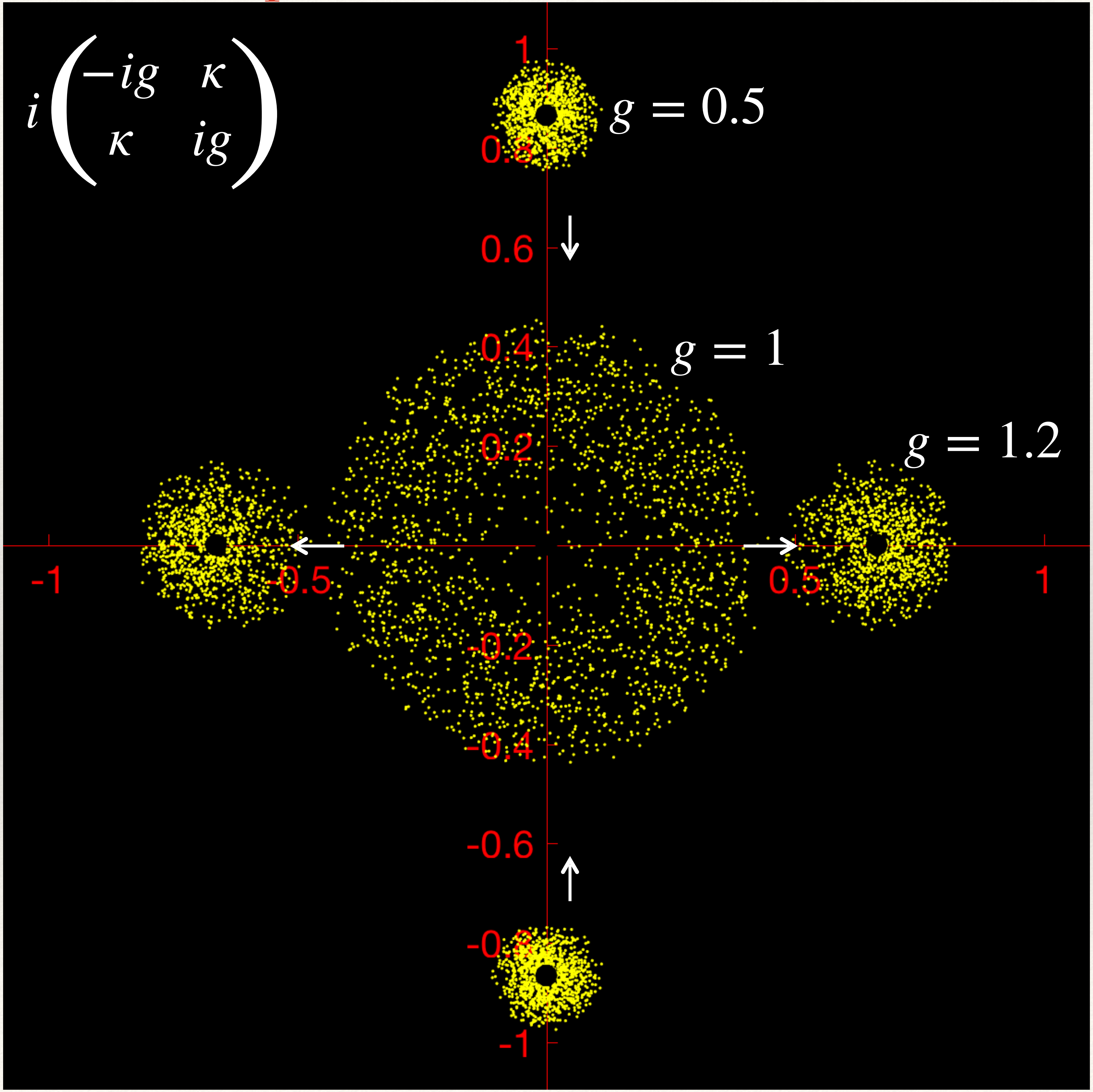
Pseudospectral radius

$$\rho_\varepsilon \equiv \max_{\mathbf{z}\in\sigma_\epsilon(H)} |\mathbf{z}|$$

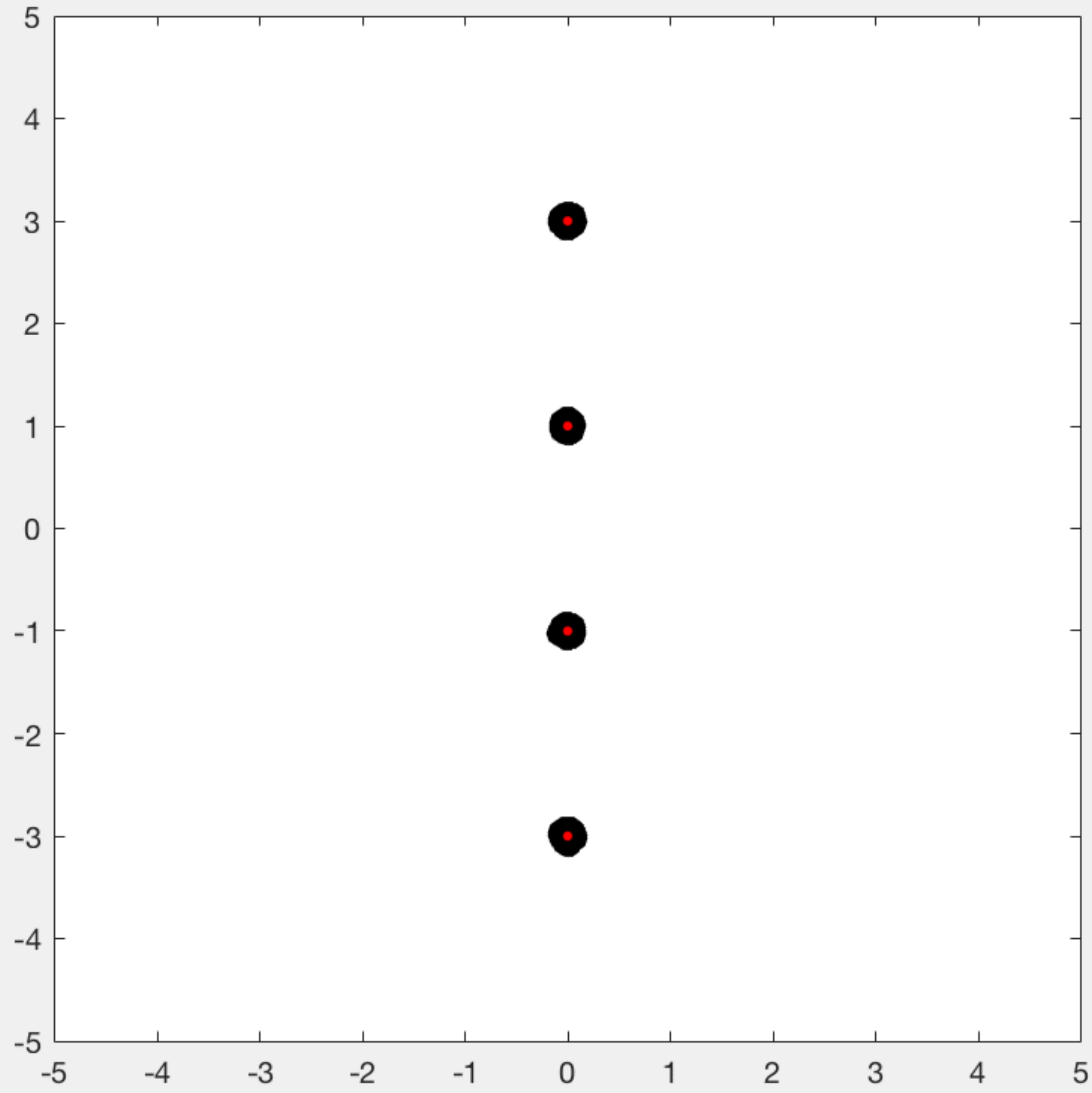
$$\sigma_\varepsilon(H) \rightarrow \underbrace{\begin{pmatrix} \delta - ig_1 & \kappa \\ \kappa & ig_2 \end{pmatrix}}_{H} + \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}$$

Non-Hermitian random matrix

Pseudospectrum of a PT-matrix with EP2



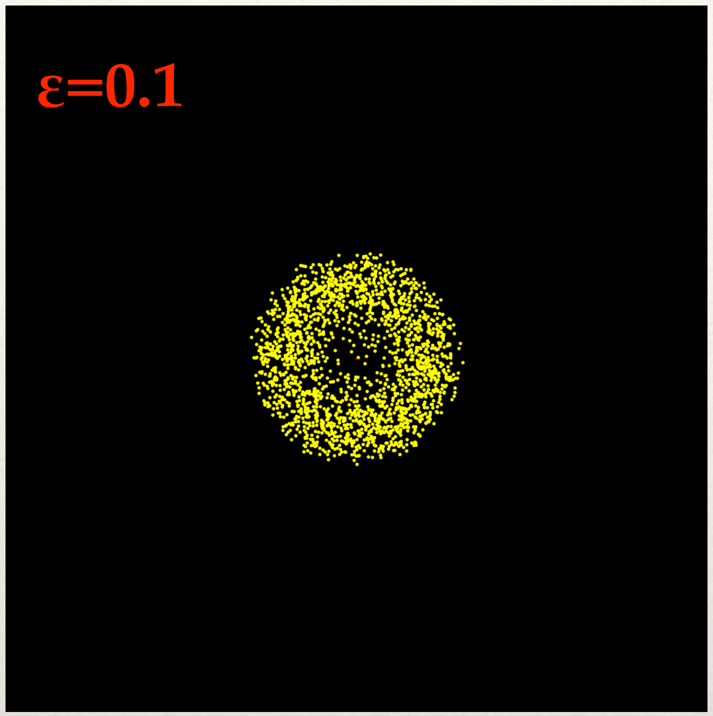
Pseudospectrum of a PT-matrix with EP4

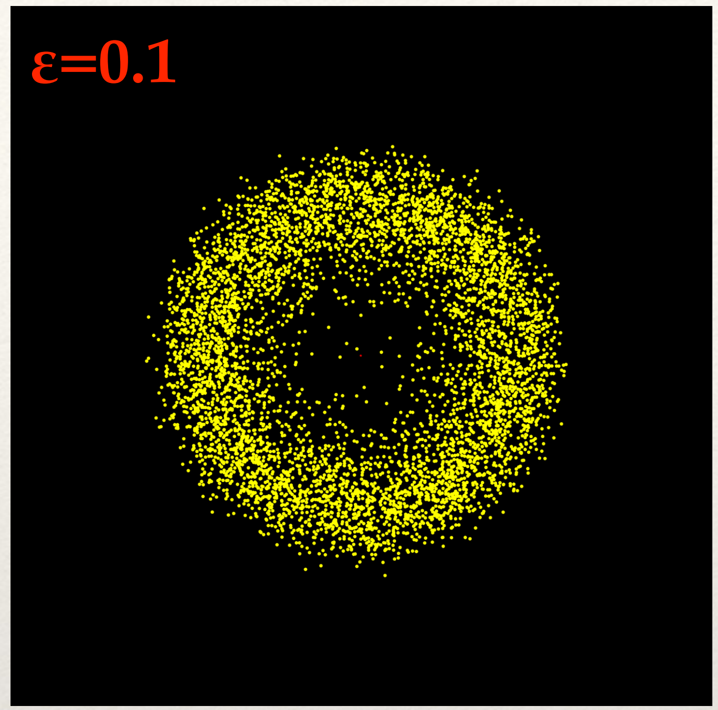


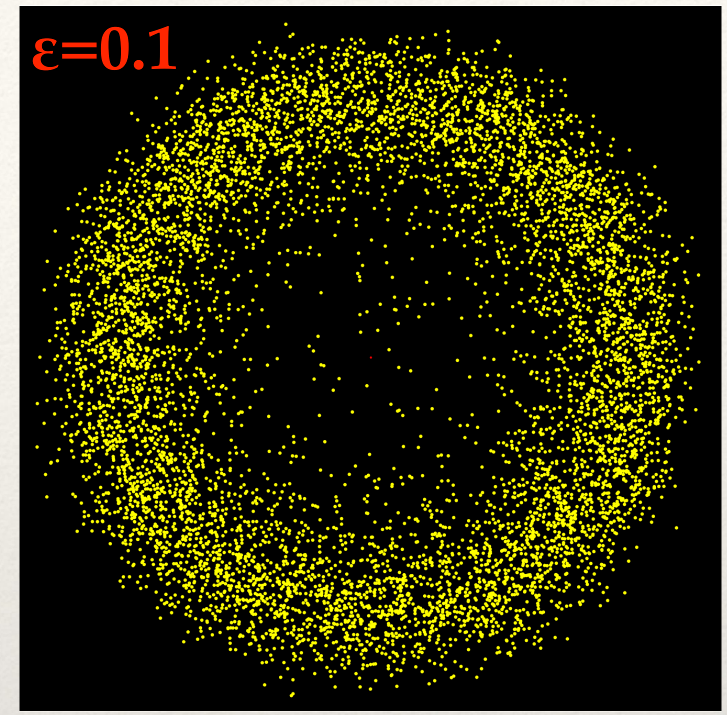
Extreme Sensitivity close to EPNs

Perturbation theory: $\Delta\omega^{EP2} = 2\kappa^{1/2}\varepsilon^{1/2}, \quad \Delta\omega^{EP3} = 1.5\kappa^{2/3}\varepsilon^{1/3}$

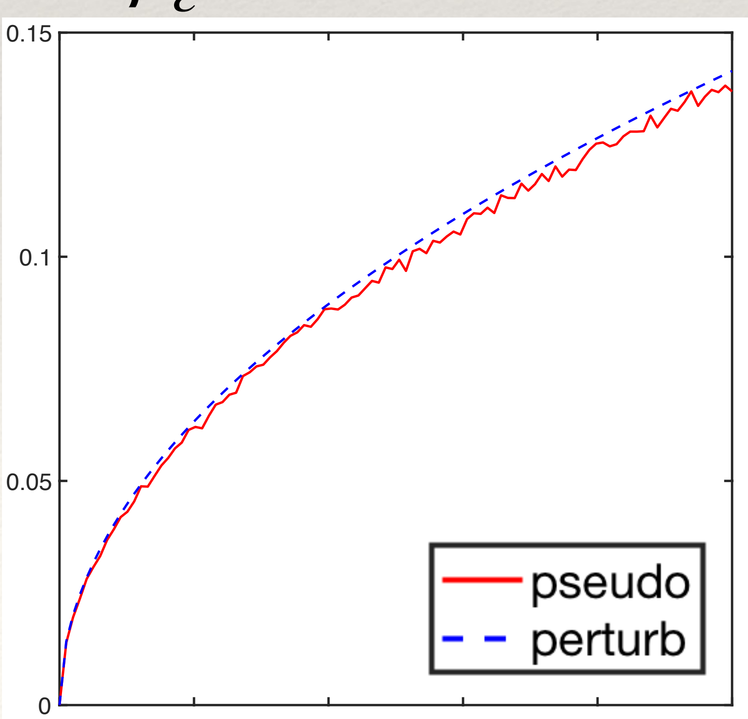
J. Wiersig, Physical Review A 93, 033809 (2016), W. Chen, et al., Nature 548, 192 (2017) H. Hodaei, et al, Nature, 548, 187 (2017).

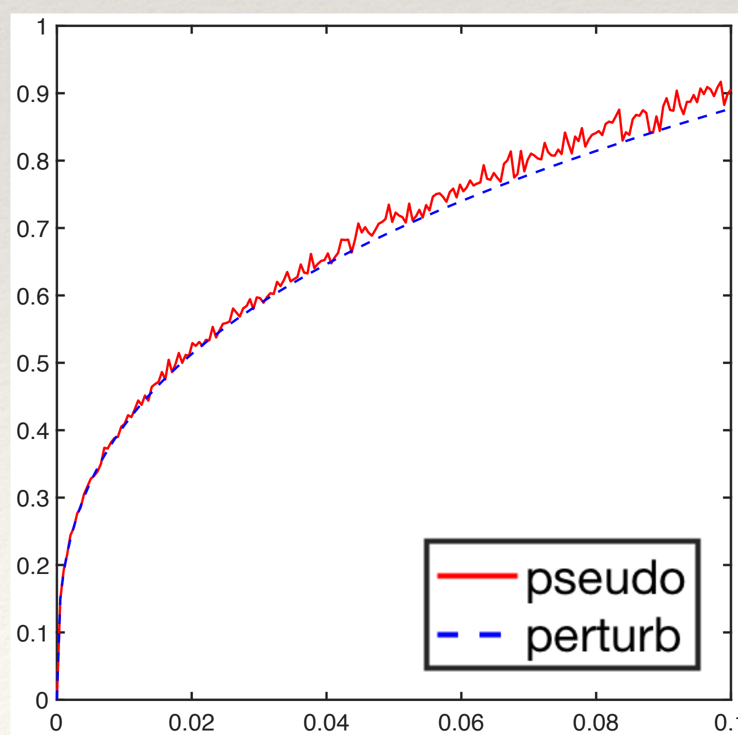


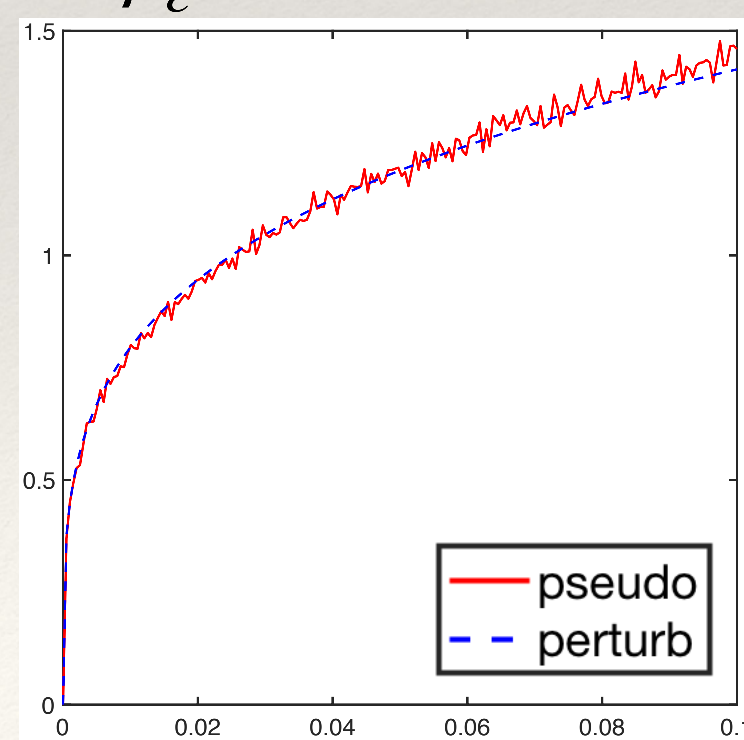




$\rho_\varepsilon \approx \epsilon^{1/2} \approx 0.3$ $\quad$ $\rho_\varepsilon \approx \epsilon^{1/3} \approx 0.46$ $\quad$ $\rho_\varepsilon \approx \epsilon^{1/4} \approx 0.56$

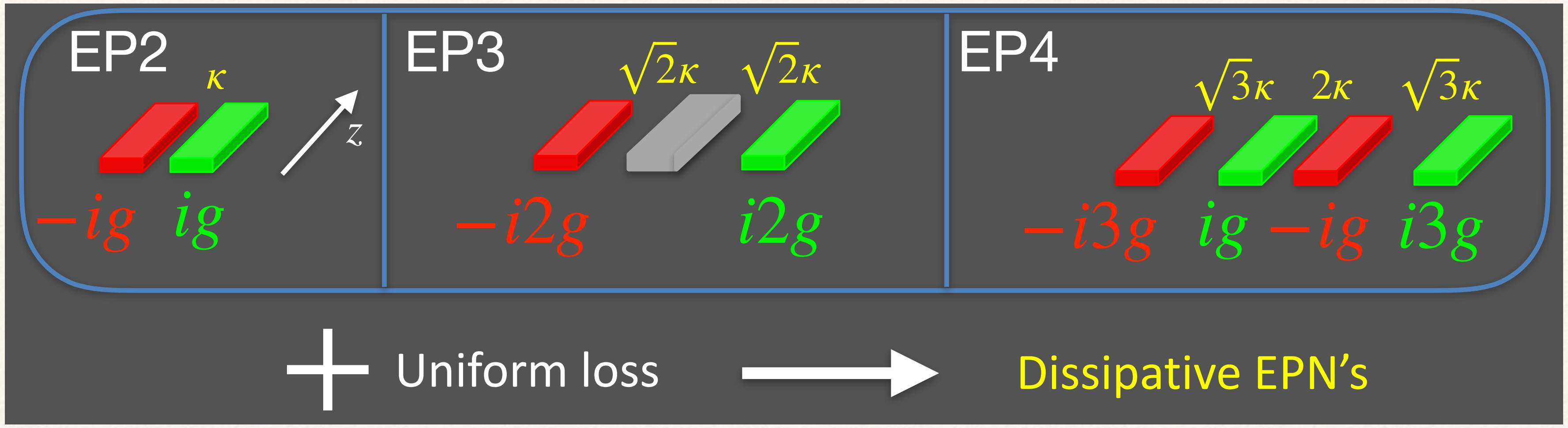






ρ_ε

Dissipative Higher order EPs

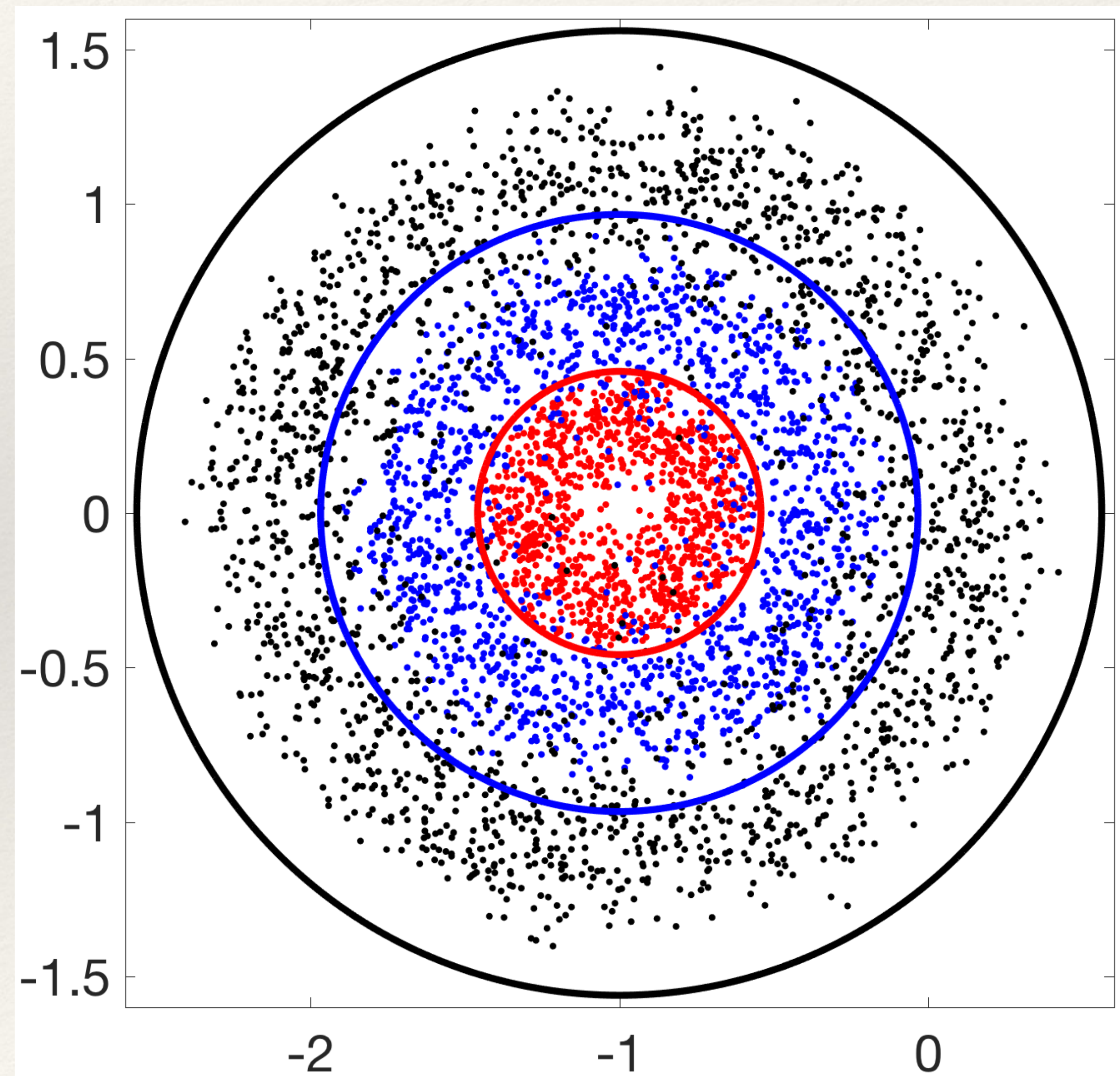


PHYSICAL REVIEW E **104**, 054218 (2021)

Transient growth and dissipative exceptional points

K. G. Makris

ITCP-Physics Department, University of Crete, 71003 Heraklion, Greece
and Institute of Electronic Structure and Laser, FORTH, 71110 Heraklion, Greece



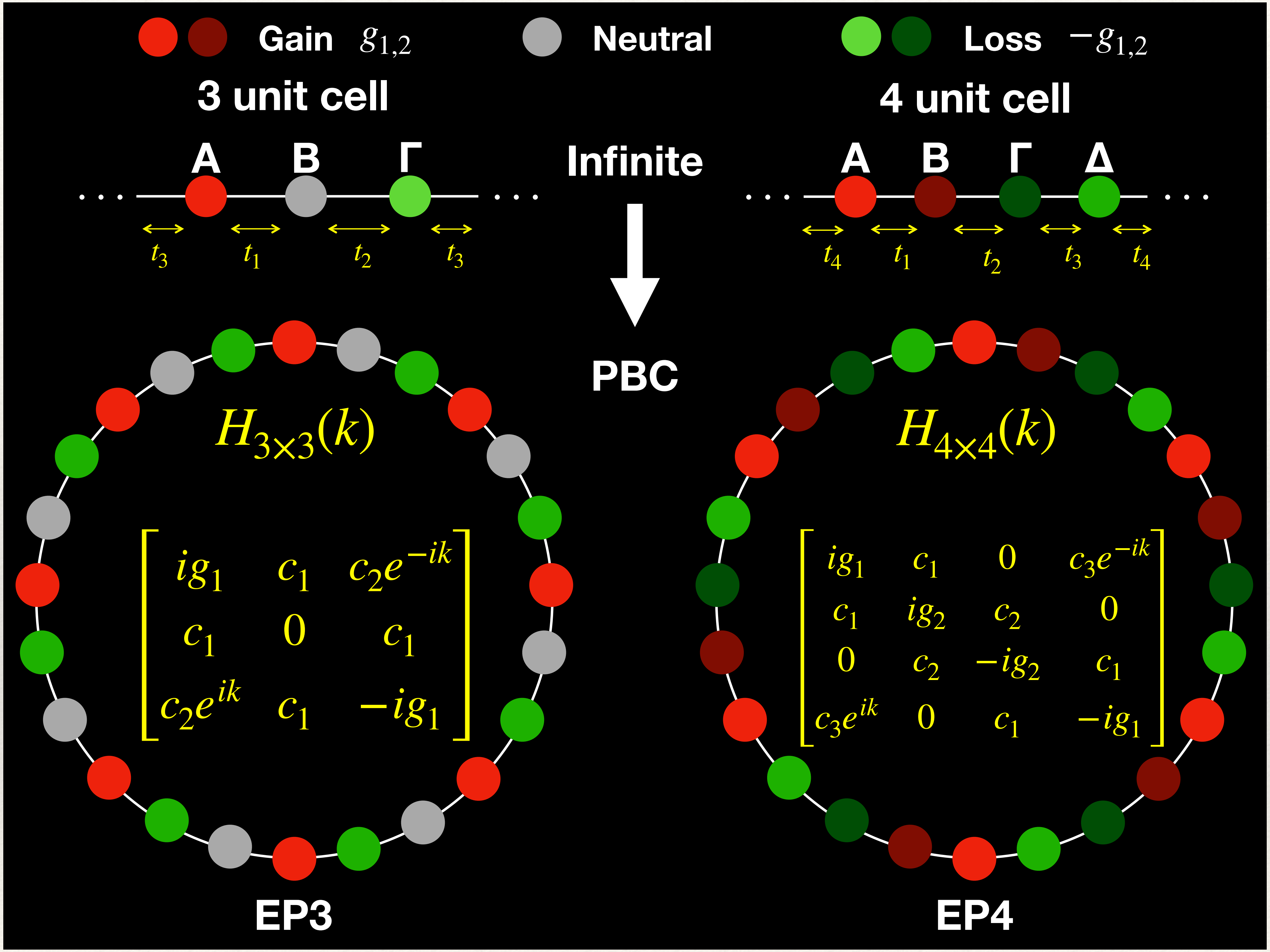
Non-perturbative approach

Pseudospectra on the Complex Plane

Part A:
Higher order Exceptional points in lattices

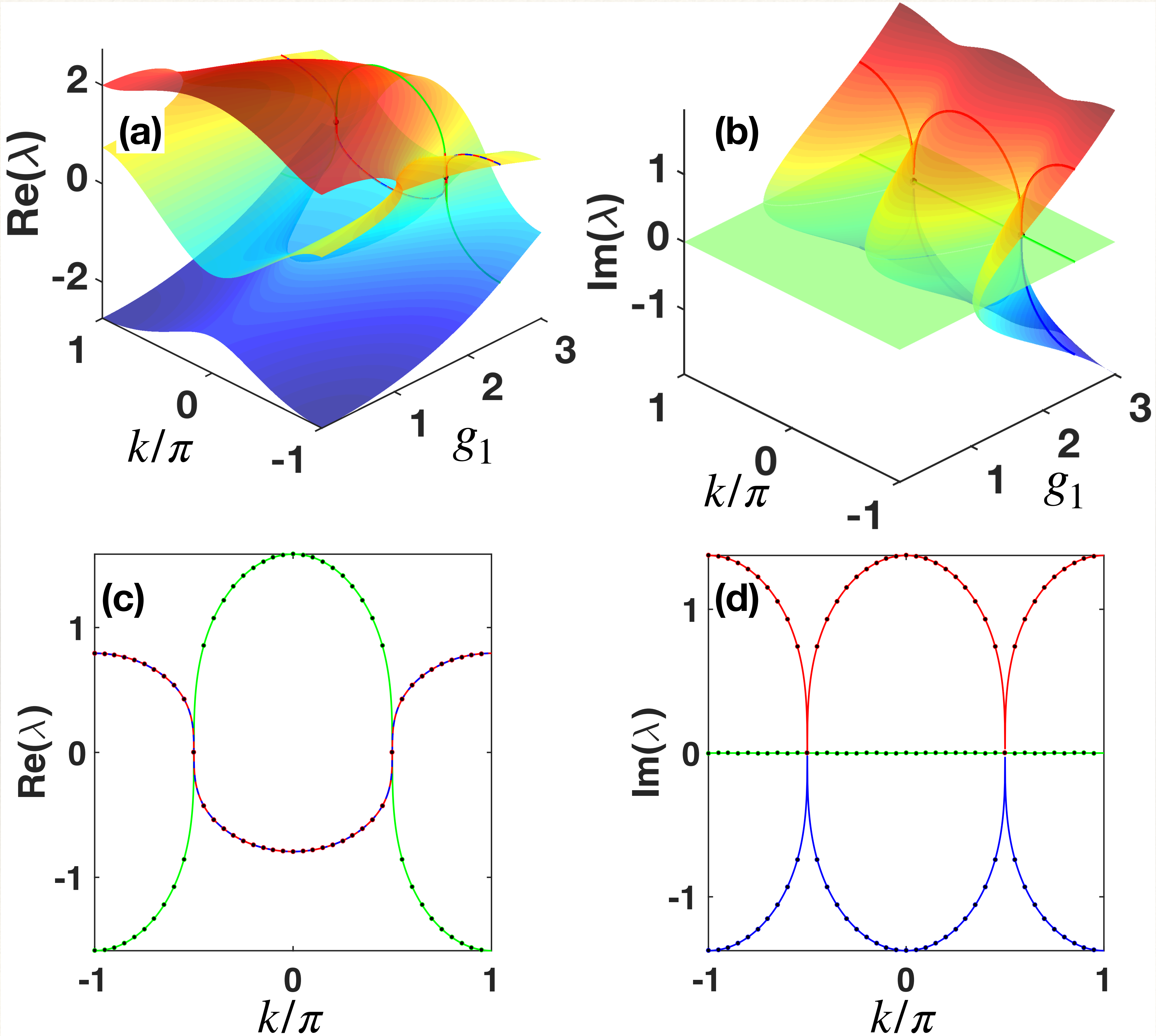
Infinite Optical lattices

HEPs in infinite optical lattices

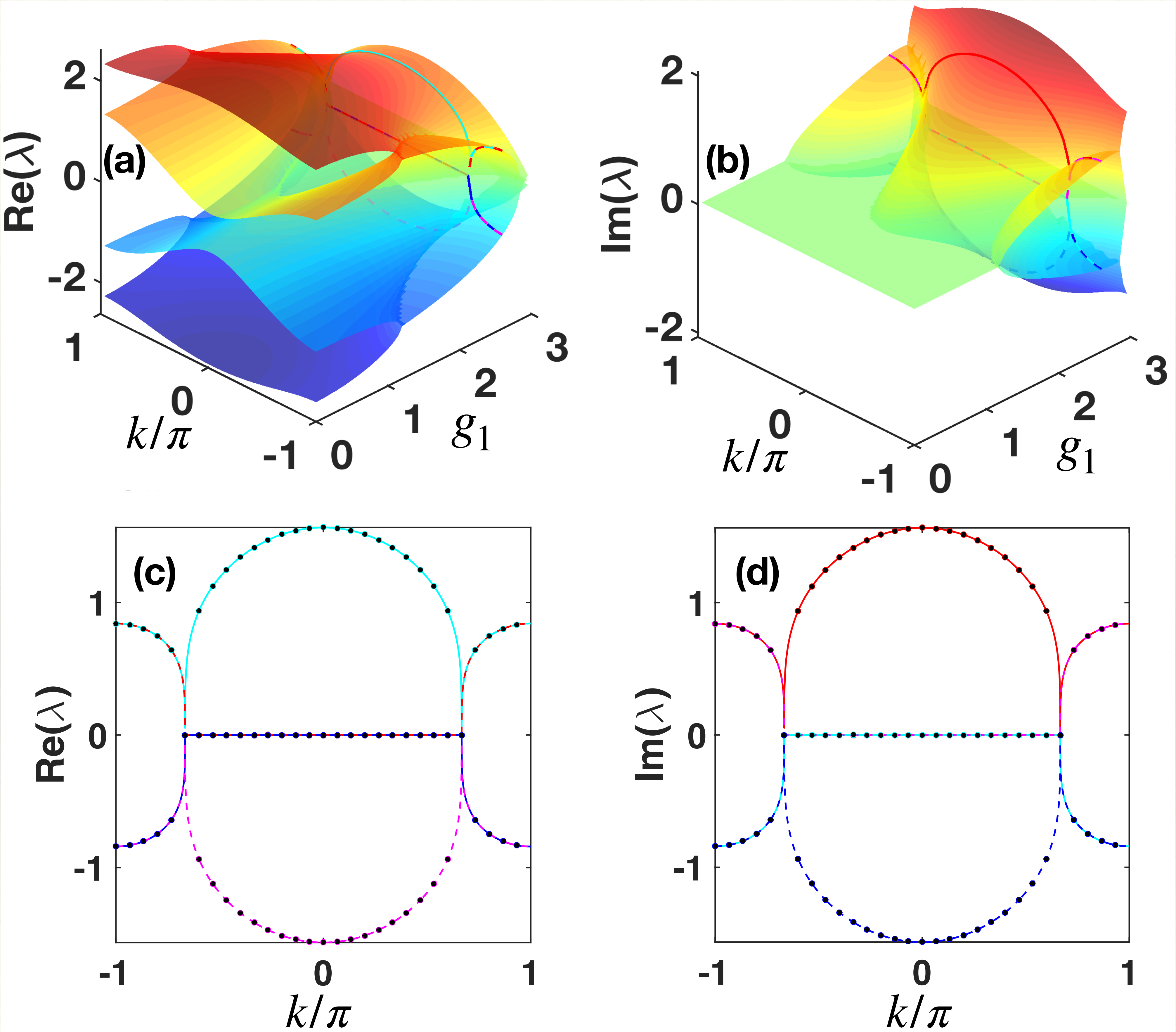


D. Kaltsas, I. Komis and K. G. Makris, Optics Letters 47,447 (2022)

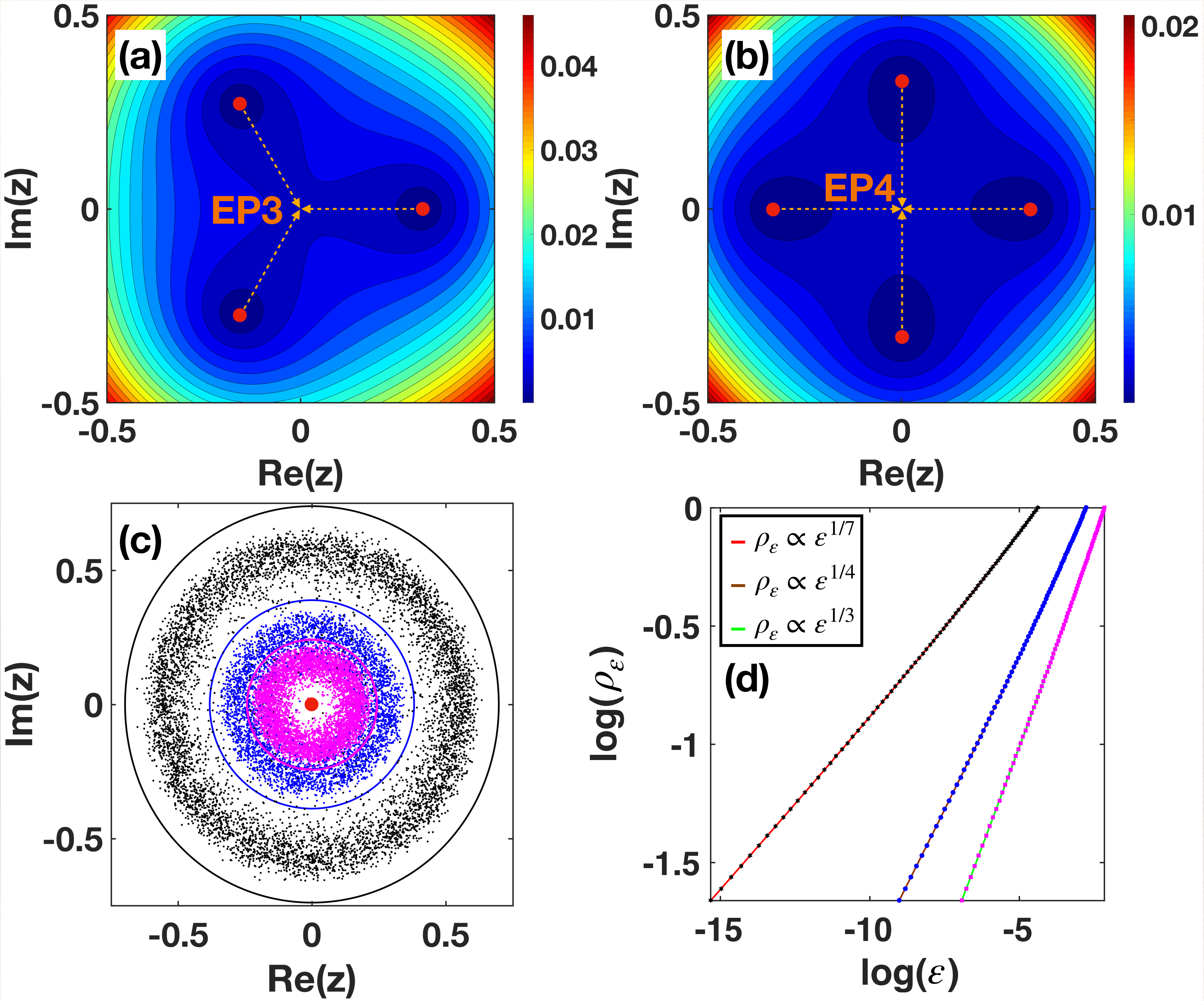
Trimerized lattice - EP3



Tetramerized lattice - EP4



Sensitivity around HEP's



D. Kaltsas, I. Komis and K. G. Makris, Optics Letters 47,447 (2022)

Part B:
Extreme behaviour around HEPs

Sensitivity vs Robustness

Complex Su-Schrieffer-Heeger Lattices

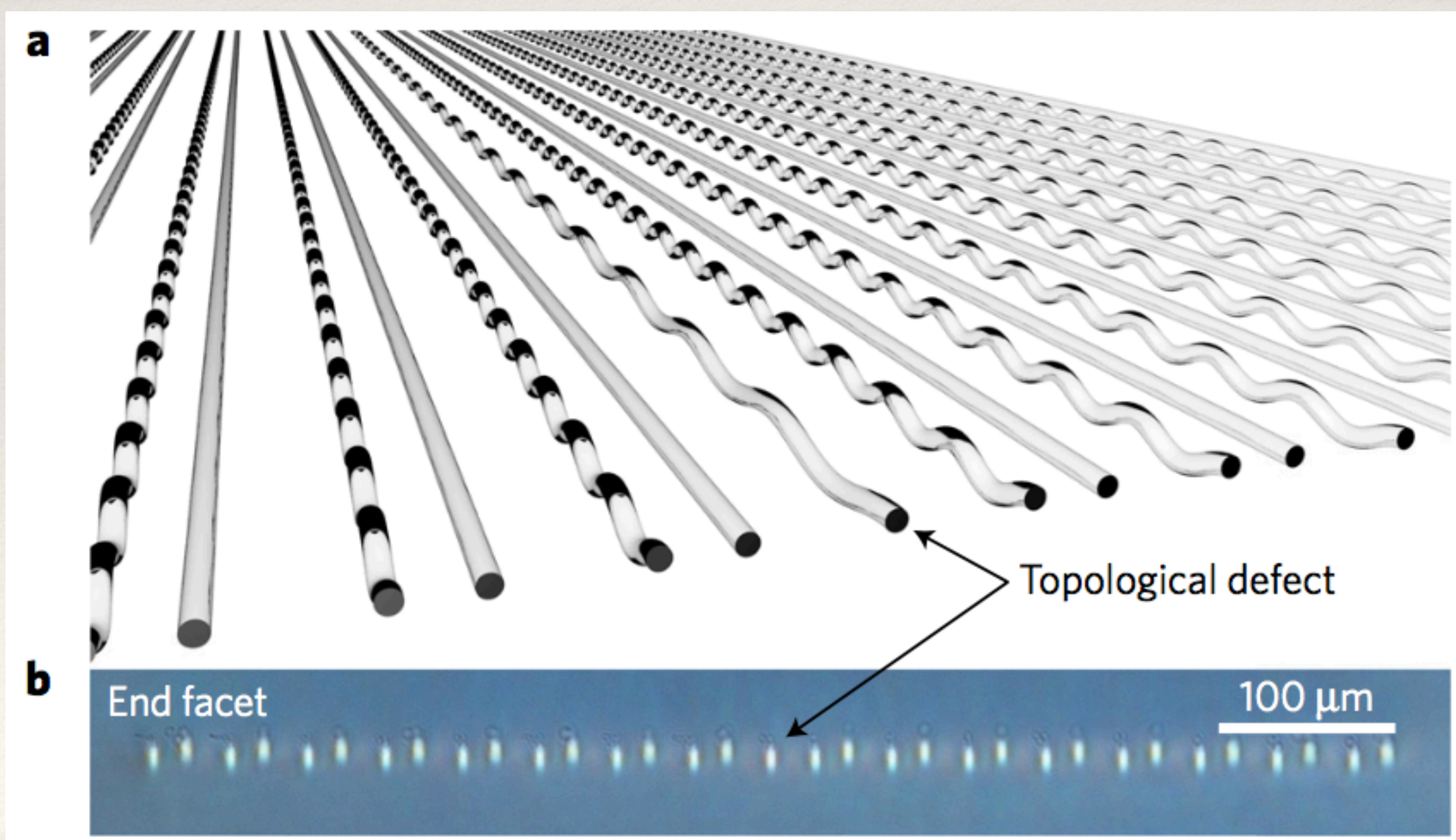
nature materials

ARTICLES

PUBLISHED ONLINE: 5 DECEMBER 2016 | DOI: 10.1038/NMAT4811

Topologically protected bound states in photonic parity–time-symmetric crystals

S. Weimann[1†], M. Kremer[1†], Y. Plotnik[2], Y. Lumer[2], S. Nolte[1], K. G. Makris[3,4], M. Segev[2], M. C. Rechtsman[5] and A. Szameit[1*]



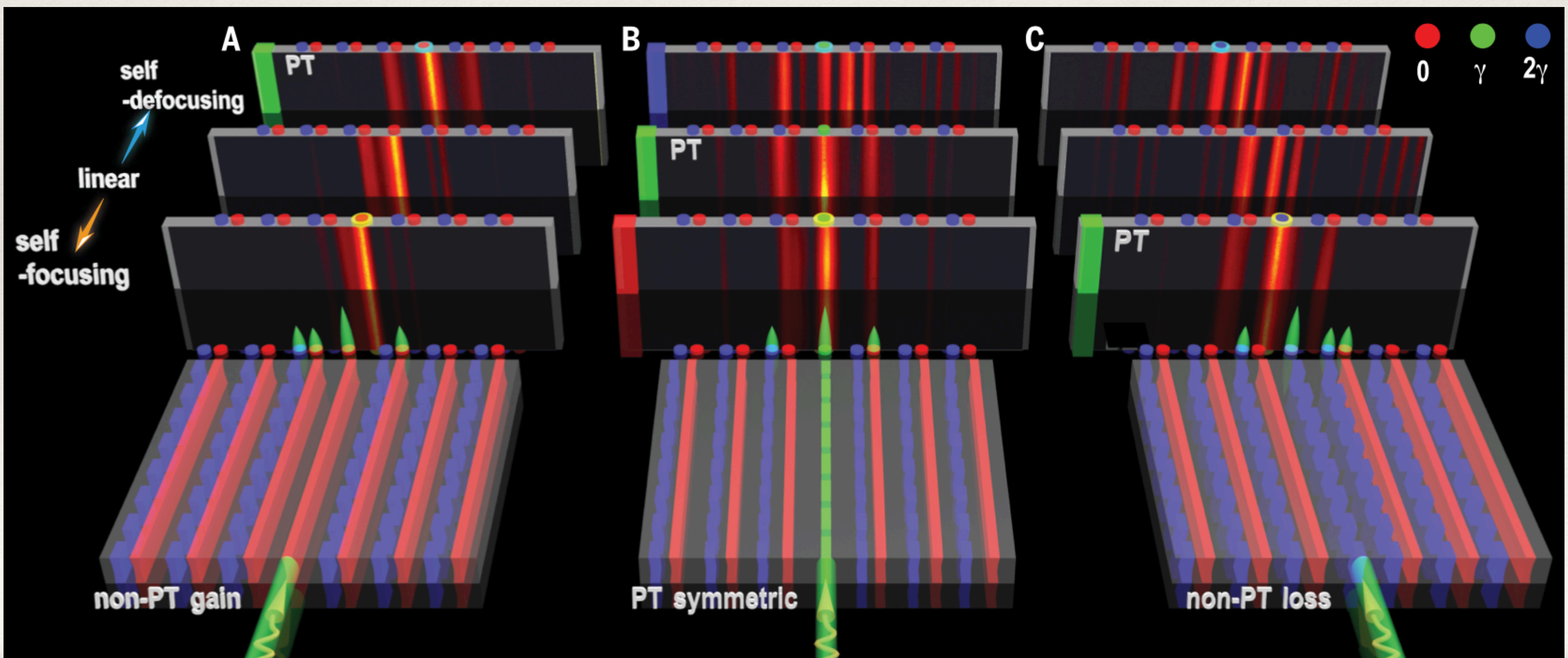
Nonlinearly induced complexity and topology

TOPOLOGICAL OPTICS

Nonlinear tuning of PT symmetry and non-Hermitian topological states

Science 372, 72 (2021)

Shiqi Xia[1]*, Dimitrios Kaltsas[2]*, Daohong Song[1]*, Ioannis Komis[2], Jingjun Xu[1], Alexander Szameit[3], Hrvoje Buljan[1,4]†, Konstantinos G. Makris[2,5]†, Zhigang Chen[1,6]†



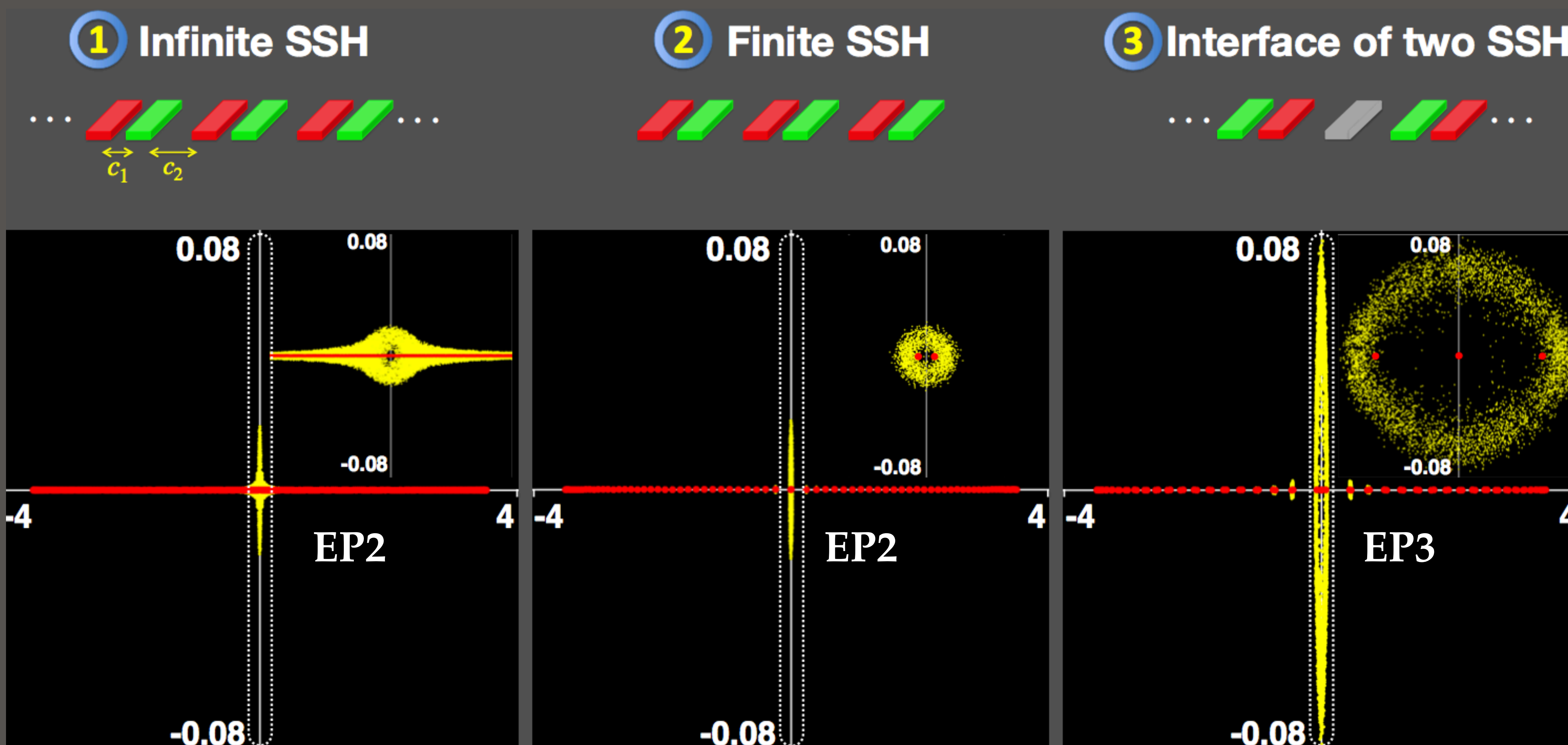
Topological Protection against Fluctuations

Robustness versus sensitivity in non-Hermitian topological lattices probed by pseudospectra

I. Komis,[1,2] D. Kaltsas,[1] S. Xia,[3] H. Buljan,[4,3] Z. Chen,[3,5] and K. G. Makris[1,2]

[1] *ITCP-Physics Department, University of Crete, Heraklion, 71003, Greece*
[2] *Institute of Electronic Structure and Laser (IESL) – FORTH, Heraklion, 71110, Greece*
[3] *The MOE Key Laboratory of Weak-Light Nonlinear Photonics, TEDA Applied Physics Institute and School of Physics, Nankai University, Tianjin 300457, China*
[4] *Department of Physics, Faculty of Science, University of Zagreb, Bijenička c. 32, 10000 Zagreb, Croatia*
[5] *Department of Physics and Astronomy, San Francisco State University, California 94132, USA*

(Dated: March 8, 2022)

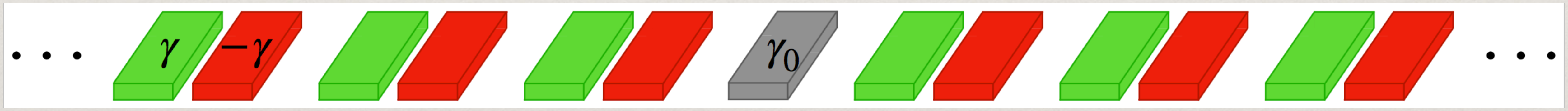


Structured Perturbations

$$\boxed{\sigma_{\varepsilon}^{str}(H) \equiv \bigcup_{j=1, E-structured, ||E_j||<\varepsilon}^{s} \sigma(H + E_j)}$$

Structured Pseudospectrum

NH- SSH interface lattice

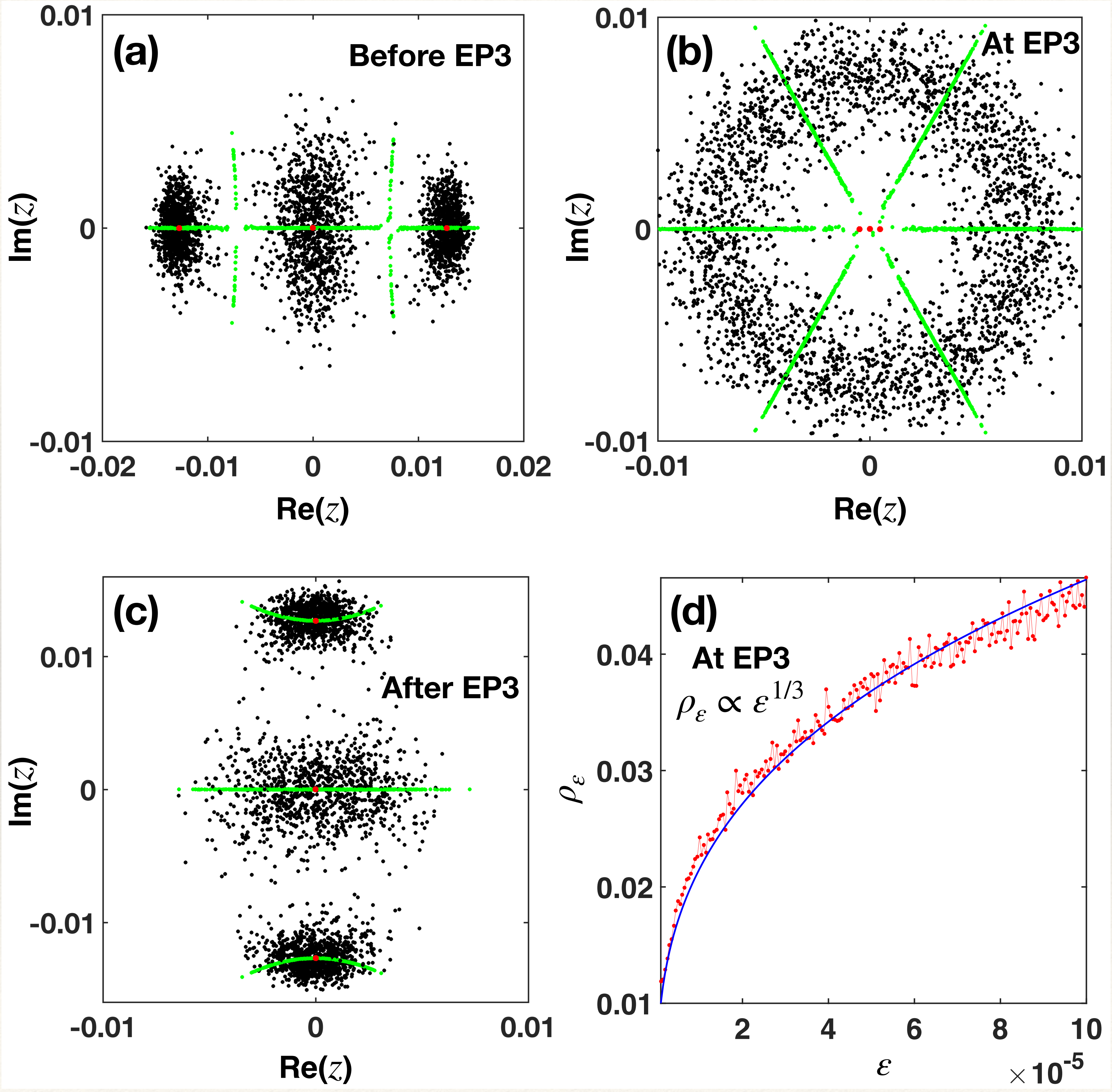


Physical Perturbations

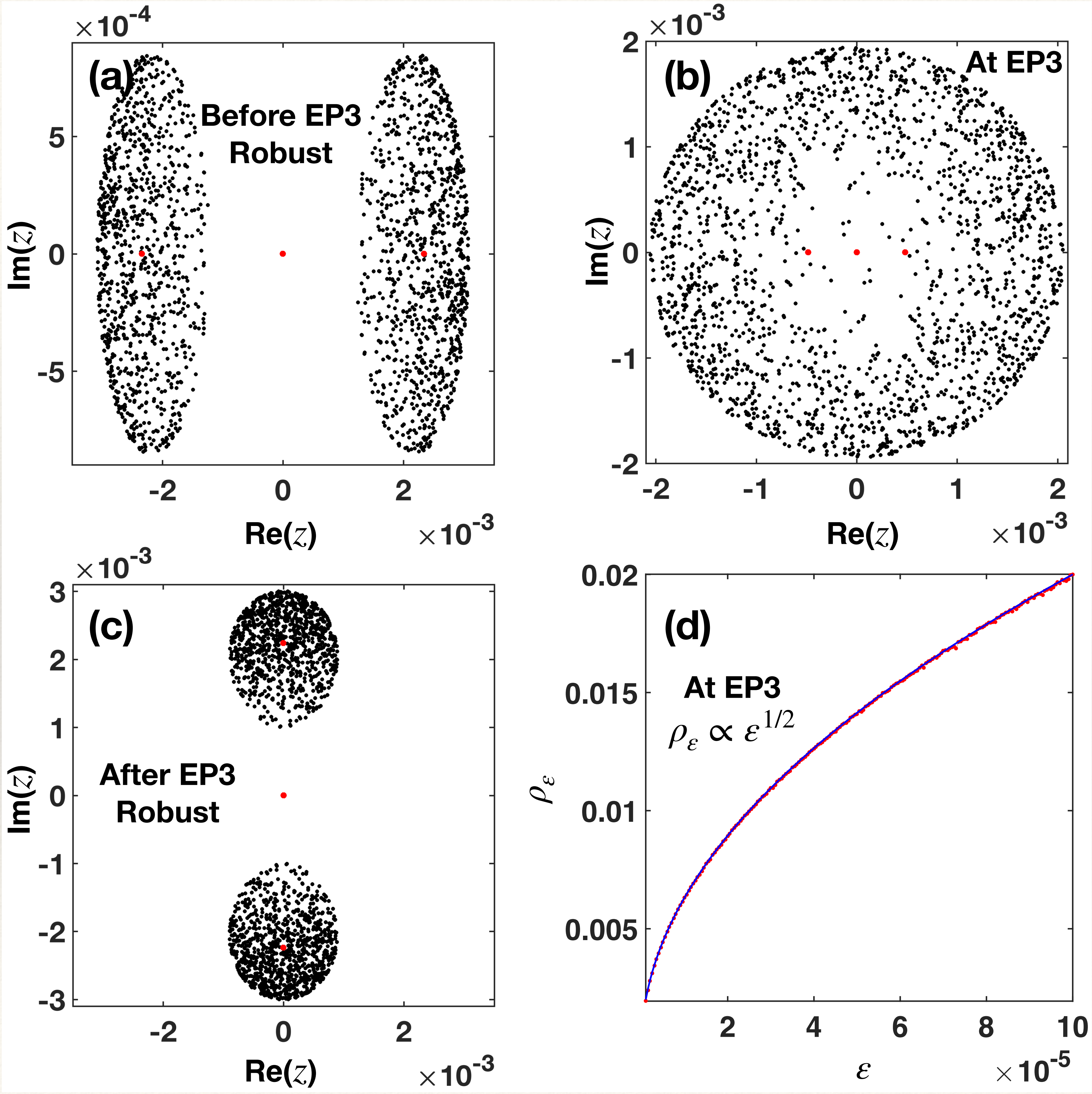
Diagonal perturbations: Change of the potential strength and the gain/loss amplitude

Off-Diagonal perturbations: Change of the coupling coefficient between waveguides

Diagonal Perturbations



Off-Diagonal Perturbations



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I. Komis

D. Kaltsas

International Collaborations

Experiment

Z. Chen
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A. Szameit
Rostock University, Germany

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Solid State Institute, Israel

M. Rechtsman
Penn State, USA

E. Rivet, H. Lissek, R. Fleury
School of Engineering, Switzerland

Theory

University of Zagreb
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Department of Physics, Croatia

D. N. Christodoulides
CREOL, USA

S. Rotter
TU-Wien, Austria

Z. H. Musslimani
Department of Mathematics, FSU, USA

Epilogue

Exceptional Point in Optics

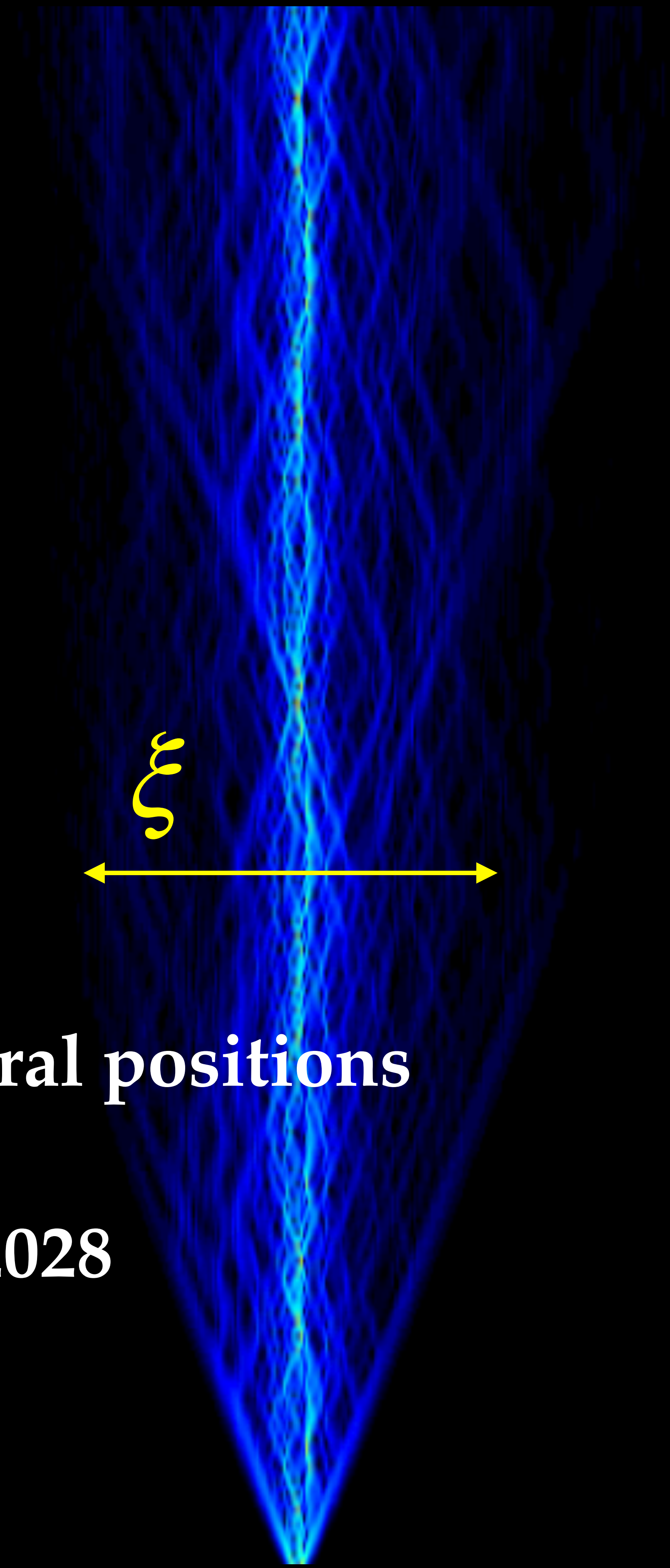
Higher order Exceptional Points in lattices

Sensitivity and Pseudospectra

Hermitian
Localization

↓

No Transport



Thank you

Open Postdoctoral positions

2023 - 2028

Beyond Anderson?